# Financial Planning Assumptions for Market-Cap Weighted and Factor-Tilted Portfolios

## Methodology Guide

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# Table of Content

# Glossary

**Beta:** In a regression analysis, beta is the sensitivity of the independent variable to fluctuations of the explanatory variable.

**Breakeven inflation:** The level of inflation that equalizes the expected returns of a real return bond and a nominal coupon bond of the same maturity.

**CAPE:** "Cyclically adjusted price-to-earnings" ratio, a measure of stock market valuation popularized by Professor Robert Shiller.

**CMA:** "Conservative Minus Aggressive", a factor that simulates a long position in the stocks of companies that invest relatively little and a short position in the stocks of companies that invest heavily. CMA is also known as the "investment factor".

**DMS:** A family of market indexes that was designed by Elroy Dimson, Paul Marsh and Mike Staunton from the London Business School, covering stock, bond and bill return over more than 120 years in over 30 countries.

**ECOC:** "Equilibrium Cost of Capital" is the historical return of the asset class over more than a hundred years, adjusted for non-recurring items such as the expansion of the price-to-dividend ratio.

**Factor-tilted:** A portfolio that incorporates the factor exposure of the DFA Global Fixed Income and the DFA Global Equity mutual funds.

**HML:** "High Minus Low", a factor that simulates a long position in high book-to-price stocks and a short position in low book-to-price stocks. HML is also known as the "value" or "relative price" factor.

**LOC:** Line of credit

**MBER:** "Market-Based Expected Return" is a measure of expected returns based on a variable that reflects the current market conditions, such as bond yields and 1/CAPE.

**Regression analysis:** A set of statistical processes for estimating the relationships between a dependent variable and one or more independent variables ("predictive variable").

**RMW:** "Robust Minus Weak", a factor that simulates a long position in high operating profits to book value stocks and a short position in low operating profits to book value stocks. RMW is also known as the "profitability" factor.

**R-Square or "R2":** The explanatory power of a regression analysis. The R-Square is located between 0 and 1. An R-Square of one means the explanatory variables of the regression explain 100% of the variations of the dependent variable.

**Shrinkage factor:** A number between 0 and 1 that is used to translate historical premiums into expected premiums. For example, a shrinkage factor of 0.7 means that the expected premium is estimated at 70% of the historical premium.

**SMB:** "Small Minus Big", a factor that simulates a long position in small-cap stocks and a short position in large-cap stocks. SMB is also known as the "size" factor.

**T-stat:** Within a regression analysis, the "t-statistic" measures whether the coefficients of a regression — for example, the beta of a simple linear regression, is statistically significant. The rule of thumb is that a t-statistic of over 2 is considered significant.

# 1. Introduction

This guide describes PWL Capital's methodology for estimating the expected returns, standard deviations, and correlations of major asset classes over a 30-year planning horizon. These parameters enable Canadian financial planners to produce financial projections for their clients. We discuss the expected risk and return for market-cap-weighted and factor-tilted portfolios. Factor-tilted portfolios are designed to replicate the factor exposure of the DFA Global Allocation funds. Since a dollar of return earned in the form of ordinary income, Canadian dividend, foreign dividend, and capital gains do not have the same after-tax value, this document addresses how we estimate the composition of expected returns. We also discuss our methodology to estimate the primary residence's expected price appreciation and standard deviation. Unless mentioned otherwise, all the data in this document is dated December 31, 2022. This data is provided for illustration purposes only. PWL publishes financial planning assumptions data updates semi-annually.

# 2. Theory of Expected Returns

## Setting Expectations

Asset allocation and financial planning decisions hinge on asset classes assumed expected return profiles. The amount of equity risk needed to achieve a goal, the sustainable spending rate in retirement, and the amount of life insurance required are some examples of critically important calculations with results that change dramatically with small changes in expected returns. The sensitivity of these calculations stems from compounding and the long-term nature of financial decisions. While their importance cannot be overstated, predicting future returns and inflation is challenging.

Investors often look to historical returns to calibrate their expectations for the future; these can be helpful but are easily deceptive. Relying on the returns of a single successful market, like the United States or Canada, results in a success bias which may result in overly optimistic expectations about the future. Another approach to setting expectations is to use the information in market prices. Popular metrics like the Shiller Earnings Yield – the 10-year trailing real earnings divided by the current price – offer a market-based expectation for real stock returns. These metrics can be helpful, but their predictive power is imperfect.

In this paper, we devise an evidence-based approach for setting expectations for the returns on stocks, bonds, bills, inflation, and housing for use in asset allocation and financial planning decisions. This paper marks an evolution in PWL Capital's methodology[1] which has been used for over half a decade. Our previous methodology used a combination of historical and market-based estimates for expected returns with an equal weight attributed to each estimate; we refer to these components as Equilibrium Cost of Capital (ECOC) and Market Based Expected Return (MBER), respectively.

Assigning a 50% weight to MBER implies that it explains 50% of the variation in future realized returns. This implication is at odds with the evidence, which varies by asset class. For example, market-based measures are highly effective at predicting bond returns but far less effective at predicting equity returns. Meanwhile, we do not find a statistically significant relationship between the breakeven rate of real-return bonds and

[1] See Bortolotti, Kerzérho, Great Expectations: How to Estimate Future Stock and Bond Returns when creating a Financial Plan, PWL Capital, 2019.

subsequently realized inflation. Therefore, our model must reflect these differences; we believe the relative weight of our market-based estimates should reflect their observed explanatory power.

Additionally, we are using long-run returns data from the Dimson, Marsh, and Staunton (DMS) data series with an adjustment for valuation changes to estimate real historical returns as the ECOC. The DMS data include failed and unsuccessful markets, which helps correct for the upward bias in estimates related to survivorship and success bias. Removing the portion of returns attributed to increasing price multiples eliminates any bias that a declining cost of capital may create as equity markets become less risky and easier to diversify compared to the earlier parts of the sample, which starts in 1901.

## Historical Returns

Predicting stock and bond returns is challenging. The historical record is the laboratory of financial economics. While it has clear limitations, historical data allow theories to be tested. For determining estimates of expected returns, historical realized returns, when examined properly, offer a reasonable starting point for what to expect in the future. The figure derived from long-run historical data is referred to in this paper as the equilibrium cost of capital (ECOC).

## (Very) Long Run Returns

The volatility of risky asset returns makes drawing insight from seemingly long periods, say 20 or 30 years, unreliable. Asness (2021) points out that while the US stock market returns beat International Developed markets by an annualized 2.1% from 1980 through 2020, nearly all that difference is explained by rising US valuations. That is, US stocks got more expensive per dollar of earnings over the period driving their returns up. Does that make the US market a better prospective investment? That seems unlikely. If anything, paying more for each dollar of future earnings is less attractive. Even with very long periods, mining more than a single country's return series is crucial. Looking at 122 years of returns for the US stock market alone may be misleading for setting future expectations due to the survivorship and success bias and rising valuations in the ex-post most successful market with 122 years of continuous history. van Binsbergen, Hua & Wachter (2022) suggest that the historical premium of US stocks over global ex-US stocks is largely explained by a combination of luck, where disasters that could have happened but did not, have boosted returns, and learning, where US valuations have risen as the market learns about their relative safety. Fortunately, there are long-run series of data for stocks, bonds, and inflation across 35 countries in the DMS database. These data include Austria and Portugal, cases of "unsuccessful" markets where equities performed very poorly, and Russia and China, two markets that failed to survive for the entire period. These data span technological transformations, wars, asset price bubbles, and financial crises, providing a complete sample of what a hypothetical US investor owning global stocks from 1901 through 2022 would have earned in real terms. The data is available in Table 1. Note that inflation is 3% for both Canada and the US over this period.

### Table 1 - Real USD Geometric Mean Returns for World and Select Country / Region Stocks and Bonds 1901 - 2022

| | Real Stock Return (USD) | Real Long Bond Return (USD) |
|---|---|---|
| Australia | 6.4% | 1.4% |
| Canada | 5.4% | 1.6% |
| Europe | 4.1% | 0.8% |
| United States | 6.2% | 1.6% |
| World | 5.0% | 1.7% |

Source: PWL Capital; Data Source: Elroy Dimson, Paul Marsh and Mike Staunton, Triumph of Optimists: 101 Years of Global Investment Returns, Princeton University Press, 2002; Elroy Dimson, Paul Marsh and Mike Staunton, Credit Suisse Global Returns Yearbook and Sourcebook, 2018, Zurich: Credit Suisse Research Institute, 2021

A nominal return of 8.1% (real return of 5.0% plus 3% US inflation) for global stocks over 122 years is nothing to complain about, but it is a problem if your financial plan requires you to earn the return of the US stock market. Over this period, US stocks delivered a nominal geometric mean return of 9.4%. This illustrates the trap of success bias. Using 9.4% (or higher based on more recent history) returns in financial planning is a bet on luck repeating itself. Readers may be interested to know that in USD terms, Australia was the best-performing stock market from 1901 through 2022, beating the US stock market by an annualized 0.12%. Is it reasonable to expect Australia to be the best-performing stock market in the world going forward simply because it has been for the past 122 years? We do not believe so.

## Decomposing Historical Returns

In the short run, stock performance is dominated by capital appreciation and depreciation. We evaluate portfolios each year to see how they have performed relative to a benchmark; almost all this variation is driven by price multiple expansion or contraction. It may be surprising, then, that the capital return has historically played a relatively minor role in long-run stock returns, as shown in Table 2. For setting expectations, it may not be reasonable to assume that the portion of returns explained by multiple expansion will repeat itself. Over the very long run, rising prices may be explained by a decreasing risk premium required by investors as markets become increasingly liquid, accessible, and diversifiable. The equity risk premium can be decomposed into several parts to isolate return components. Dimson, Marsh, and Staunton (2007) demonstrate the equity risk premium decomposition from 1900 through 2005 for global stocks, as shown in Table 2.

### Table 2 - World Equity Premium Decomposition 1900 - 2005

| | Real Dividend Growth Rate | plus Expan-sion of the P/D Ratio | plus Geomet-ric Mean Div-idend Yield | minus US Real Interest Rate | equals Equity Premium for US investors |
|---|---|---|---|---|---|
| World Equities | 0.77% | 0.68% | 4.23% | 0.96% | 4.72% |

Source: Elroy Dimson, Paul Marsh and Mike Staunton, The Worldwide Equity Premium: A Smaller Puzzle (April 7, 2006). Chapter 11 of R Mehra (Ed), Handbook of the Equity Risk Premium. Elsevier, 2008, pages 467–514, AFA 2008 New Orleans Meetings Paper; EFA 2006 Zurich Meetings Paper

The dividend yield has historically been the biggest driver of long-run returns, while multiple expansion has played a relatively minor role. We have updated the multiple expansion figure for the global market index for the period ending December 2022 and found that it has decreased slightly, from 0.68% to 0.47%. Removing the 0.47% multiple expansion from the 5.07% real historical geometric mean global stock return, we arrive at an adjusted historical real return of 4.60% for the world equity index. This historical figure for the world index is our ECOC foundation for building expectations for each country. The alternative of using a particular country's historical return as the foundation of its expected return results in the potential for success or unsuccess biases clouding the estimate.

Bond returns similarly have a capital return and an income return. Over the very long period under examination, interest rates have done a round-trip rise and fall. We need a precise breakdown of the contribution from capital and income return for bonds globally, similar to the one seen for stocks in Dimson, Marsh, and Staunton (2007). Still, based on the round trip of interest rates starting low, rising, and falling globally over the entire period, we estimate that the capital return is negligible in estimating future returns. Based on this, our ECOC for fixed income is simply the long-run 122-year return for long-dated government bonds. This is imperfect since we are estimating returns for bond portfolios closer to an intermediate maturity on average and including corporate bonds. Using the long-term government bond return, we implicitly assume that the term premium is making up for the lack of a credit premium in our estimate.

## Economic Variables and Predictability

With the ECOC starting point, the next question is whether expected returns are constant or time-varying. If they are constant, we should simply use ECOC as our estimate for future returns; if they are time-varying, we should look for economic variables that predict differences in expected returns. It is abundantly clear from the literature and economic logic that expected returns change over time[2]. For example, investors require a higher premium for owning risky assets when the market is riskier or when investors are more risk-averse. It is, however, less clear that economic variables reliably predict these differences in a way that can be used in making financial decisions.

## Risk or Behavior?

There are two possible theoretical cases for predictable stock returns. In the behavioral case initiated by De Bondt and Thaler (1985), investors overreact to news, causing prices to move above or below their "fair value," to which they must inevitably return. The herding behavior of investors can push this effect further, creating a feedback loop. This is the case for asset "bubbles" which must eventually pop. In the risk-based case, investors respond rationally to economic circumstances by adjusting their required return for holding risky assets. If predictability is behavior-based, and returns are predictable, there is alpha (excess risk-adjusted returns) to be earned by savvy investors. On the other hand, if predictability is risk-based, investors may expect to earn higher returns when risk premiums are elevated, but they are doing so by taking more risk. It is not possible to definitely determine which case is true.

[2] See, for example, Business Conditions and Expected Returns on Stocks and Bonds by Fama and French (1989)

## Predictability in the Literature

The literature on predictability is mixed. Campbell and Thompson (2008) find evidence of return predictability. Goyal and Welch (2008) and Goyal, Welch, and Zafirov (2023) find that most predictability models would not have helped an investor with access only to available information to profitably time the market, further driven home by Asness *et al.* (2017) with the simple explanation that observed predictability using an entire data series has an inherent hindsight bias; real-time investors do not know what future valuations will be, so any notion of a low or high valuation in the historical data may not tell us much about whether current valuations are low or high relative to future valuations. Dimson *et al.* (2013) similarly find that observations of predictability can create something like the "Gambler's Fallacy": the belief that deviations from expected behavior are likely to be followed by deviations in the opposite direction. Within the confines of an ex-post observed trendline in stock returns, the mean reversion seems blatantly obvious; what is not obvious is that the trend line will continue. The authors find empirically in the global data from 1900 through 2012 that "for investors who do not have perfect foresight and who do not know the parameters of the model for the distant future, there is no consistent relationship between forecasts and outcomes. Moreover, for cases with a marginally significant relationship, roughly as many countries are significantly negative and positive. Cochrane (2007) explains that the out-of-sample test used by Goyal and Welch (and repeated by Dimson *et al.*) is an interesting diagnostic, but it is not a statistical test of predictability. Still, their out-of-sample tests are an important caution about using return forecasts to form aggressive market-timing portfolios given currently available data. Present value logic implies that if both returns and dividend growth are unforecastable, the price/dividend ratio is constant. Empirically, this is not the case, as the price/dividend ratio is highly volatile. The question that Cochrane asks is: "how much of dividend growth or returns is forecastable?" Historically high prices (low dividend yields) have been resolved by subsequent low returns, not by higher dividend growth which implies that returns are forecastable while cash flows are not.

Compounding the issues within this debate is that predictable returns do not necessarily make stocks safe investments in the long run. Even if there is mean reversion in stock returns, Pástor and Stambaugh (2012) explain that investors today do not know the long-term mean to which returns will revert; investors do not know what the equity risk premium is today; and investors do not know the values of the parameters of the return-generating process. It is safe to say that returns are not predictable in a way that can be used to time the market profitably, but it is difficult to reject the null hypothesis that expected returns vary through time.

## Combining Historical (ECOC) and Predictive (MBER) Expected Return Estimates

Given the evidence, we are hesitant to completely adopt or ignore the possible effects of predictability on expected returns in developing a point estimate. We are acutely aware of the financial planning implications of varying expected return assumptions as the market changes. Financial planning decisions involving saving, spending, and long-term asset allocation may be affected by the expected return assumption being used. For example, suppose the CAPE in a country is high (implying lower expected returns). In that case, an investor may save more, spend less, or implement a more aggressive asset allocation to meet their expected return requirement. If CAPE is sufficiently predictive, these changes are sensible. Davis (2015) finds, using bootstrap simulations with varying degrees of pre-defined predictability, that at an $R^2$ of 0.30, the investor varying their asset allocation each year based on the 10-year CAPE forecast would beat the buy-and-hold investor in only 18% of simulations, even under the fixed and known return forecasting parameters in the model; live return forecasting parameters are unlikely to be fixed, and they are unknown. However, where evidence of long-term return predictability exists, more is needed to facilitate successful market timing.

As a middle ground, we give some weight to market-based expected returns without relying on them entirely. Our approach to assigning weights to historical and predictive expected return estimates starts with the historical predictive power of the selected variables. We similarly define that variable's predictive power in our return-generating process by assigning a weight to the predictive expected return estimate.

# Predictive Power of Predictive Metrics

We use linear regression between our market-based measure of expected return and the actual realized return during the following ten years using US data from Robert Shiller's data series for bonds and stocks. We use US data due to its availability over long horizons. As Dimson, Marsh, and Staunton (2013) pointed out, global data will provide different results. However, our objective is not to build a model perfectly rooted in global history, but to create a plausible model for the future.

The regression results provide a coefficient of determination, $R^2$, measuring the variation in the returns explained by the predictive variable, and a regression coefficient, beta, measuring the sensitivity of returns to the predictive variable. For example, if a 1% change in 1/CAPE predicts a 1% change in returns in the following decade, the beta of the regression is precisely 1. We use both overlapping and non-overlapping samples. Overlapping samples, often used in financial analysis, overstate t-statistics while only providing marginal benefit, as described in Boudoukh *et al.* (2019). Non-overlapping samples limit us to seven 20-year samples in the historical data. Using both lenses together is helpful.

Table 3 provides regression results based on the data from Robert Shiller. This dataset is the longest series available, starting in 1871. In this exercise, we measure the ability of long-term US government bond yields to predict their future 20-year nominal annualized returns and the ability of 1/CAPE to predict future 20-year real annualized returns of the S&P 500. This analysis reveals that long bond yields have a high predictive efficacy (high R-squared, beta and t-statistic) and 1/CAPE has a low but significant predictive efficacy (lower R-squared, beta and t-statistic).

## Table 3 - Regression Results (Shiller data) 1871-2022

| | $R^2$ | Beta | t-statistic |
|---|---|---|---|
| Long-Term US Treasury Yields vs. Nominal Returns (overlapping) | 0.84 | 0.95 | 92.0 |
| Long-Term US Treasury Yields vs. Nominal Returns (non-overlapping) | 0.75 | 1.03 | 5.0 |
| US Stock 1/CAPE vs. Real Returns (overlapping) | 0.38 | 0.61 | 29.7 |
| US Stock 1/CAPE vs. Real Returns (non-overlapping) | 0.32 | 0.41 | 2.5 |

Source: PWL Capital; Data Source: Robert Shiller

To corroborate the predictive ability of market-based metrics, we analyze the Ibbotson Associates dataset, which spans a shorter period (1926-2022), but provides additional information about US Treasury bills and intermediary term US government bonds. Our results are presented in Table 4.

## Table 4 –Regression Results (Ibbotson Data) 1926-2022

| | R2 | Beta | t-statistic |
|---|---|---|---|
| US 1-Month T-Bill Yields vs. Nominal Returns (overlapping) | 0.33 | 0.46 | 21.3 |
| US 1-Month T-Bill Yields vs. Nominal Returns (non-overlapping) | 0.74 | 0.81 | 13.9 |
| Interm.-Term US Treasury Yields vs. Nominal Returns (overlapping) | 0.71 | 0.77 | 47.8 |
| Interm.-Term US Treasury Yields vs. Nominal Returns (non-overlapping) | 0.63 | 0.73 | 3.5 |
| Long-Term US Treasury Yields vs. Nominal Returns (overlapping) | 0.90 | 1.16 | 93.2 |
| Long-Term US Treasury Yields vs. Nominal Returns (non-overlapping) | 0.92 | 1.06 | 7.3 |
| US Stock 1/CAPE vs. Real Returns (overlapping) | 0.29 | 0.91 | 25.1 |
| US Stock 1/CAPE vs. Real Returns (non-overlapping) | 0.32 | 0.41 | 2.5 |

Source: PWL Capital; Data Source: Morningstar

We draw three observations from Table 3 and Table 4:

1. The Ibbotson data confirms the high predictive efficacy of bond yields, and the low (but significant) efficacy of CAPE yields we found in the Shiller data.
2. Intermediate-term bond yields (with an average maturity of 5.5 years) appear less effective than long-term Treasury bond yields (with an average maturity of 20 years) in predicting future 20-year returns (long bond regressions have higher R-squared and betas).
3. Finally, US Treasury bill yields appear to have even less predictive efficacy than CAPE yields. T-bills have slightly higher overlapping R-squares, far lower betas and a similar R-squared compared to CAPE yields.

To incorporate these findings into our expected returns model, we take approximately the product of the $R^2$ and the beta of the overlapping regressions as the weight of the market-based component of the model. Recognizing that this is not an exact science, we are assigning a weight of 75% to the MBER component for fixed income returns, 25% for equity returns, and only 15% for cash instruments. The remainder in all cases comes from the 122-year survivorship-adjusted and valuation-change-adjusted historical average or ECOC. Section 9 provides more details about expected returns computation.

# 3. Theory of Factor Premiums

## Estimating Factor Risk Premiums

A market capitalization-weighted investor expects to earn the equity risk premium – the premium of stock returns over less-risky treasury bills – for taking on the risk of stock ownership. Since the 1980's research in financial economics has identified other risks for which investors seem to expect compensation for bearing. According to research, there may be more than 400 of these systematic pricing factors (Harvey & Liu, 2019), commonly referred to simply as "factors," documented in published literature. The proliferation of factors was referred to as a "factor zoo" by the then-president of the American Finance Association John Cochrane, in his 2011 presidential address. In a multi-factor world, investors may structure their portfolios to capture more than a single risk premium.

## Taming the Factor Zoo

Given the zoo of factors and the replication issues across many scientific fields, a fair question is whether factors are robust out-of-sample or impossible to replicate due to in-sample data mining. Jensen, Kelly, & Pedersen (2021) analyze 153 factors across 93 countries using a Bayesian framework which is effective for making reliable inferences in the face of multiple testing. The framework starts with the prior belief that factors have zero expected return and allows the in-sample results to increase the estimated premium incrementally. The authors find that most factors in their sample can be replicated in-sample, organized into 13 themes, work out-of-sample, and are strengthened (not weakened) by the large number of observed factors. They additionally show that some out-of-sample decay should be expected in light of Bayesian posteriors based on published evidence. Stated simply, starting from the prior belief that a factor premium is not different from zero, a published observation strong enough to update this belief may be partially due to luck in which case some of the out-of-sample premiums are expected to decay. Based on this Bayesian perspective, lower out-of-sample factor premiums are precisely what we would expect to see. Jensen, Kelly, and Pedersen (2021) find a decline of about a third in post-publication premiums. McLean and Pontiff (2015) study the out-of-sample and post-publication return predictability of 97 variables shown to predict cross-sectional stock returns and find that portfolio returns are 26% lower out-of-sample and 58% lower post-publication. They estimate a 32% (58%–26%) lower return from publication-informed trading.

## Choosing Factors

Factor premiums stand up to scrutiny in the data, though we should expect a post-publication decline in the premium of approximately a third. We still need to identify which factors in the zoo we want to pursue in designing investment portfolios. Fama and French (2018) test six possible factor models based on the maximum squared Sharpe ratio and find that a model including the market risk factor (Mkt), the small-cap factor (SMB), the value factor (HML), the profitability factor (RMW), the investment factor (CMA), and momentum factor (UMD) performs well in all tests. They warn that while momentum appears to perform well in the model, deviating too far from theory should be approached with caution. Fama and French (2015) offer the dividend discount model as a theoretical anchor for the factors in their Five-Factor model, but the strong performance of momentum in tests still poses a problem for portfolio construction. Despite its challenging theoretical story, momentum is difficult to ignore empirically. However, Detzel, Novy-Marx, and Velikov (2021) find that when transaction costs are considered, high-cost-to-implement factors like momentum perform worse than the Fama and French Five-Factor model.

## The Fama and French Five-Factor Model

The dividend discount model says that the theoretical value of a share of stock is the discounted value of expected dividends per share at the infinite horizon.

$$m_t = \sum_{\tau=1}^{\infty} E(d_{t+\tau})/(1+r)^{\tau} \quad (1)$$

Equation 1 shows that the price $m_t$ at time $t$ is equal to the expected future dividends per share, $E(d_{t+\tau})$, discounted at the long-term average expected stock return $r$.

One of the problems with the dividend discount model is that not all firms pay dividends. Miller and Modigliani (1961) show that given investment policy, dividend policy is irrelevant to the valuation of shares. With dividend policy irrelevance, the value of expected dividends equals expected earnings minus expected investment. According to Miller and Modigliani (1961), the total market value of the firm's stock is given by Equation 2.

$$M_t = \sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau} \quad (2)$$

$Y_{t+\tau}$ is the expected earnings and $dB_{t+\tau}$ is the expected change in book equity (asset growth). Scaling both sides of Equation 2 by the book value of equity, $B_t$, Equation 3 gives the theoretical valuation equation as presented by Fama and French (2015).

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}}{B_t} \quad (3)$$

This theoretical valuation equation makes three statements about expected stock returns:

1. If we hold everything in Equation 3 constant except for the market value of the stock, $M_t$, and the expected stock return, $r$, then a lower ratio of $M_t/B_t$ must imply a higher expected stock return. All else equal, a company with a lower price must have a higher discount rate. This is an expression of the value premium.
2. If we hold everything in Equation 3 constant except for expected future earnings, $Y_{t+\tau}$, and the expected stock return, $r$, then higher expected earnings must imply a higher expected stock return. All else equal, if two companies trade at the same relative price, the company with higher profits must have a higher discount rate. This is an expression of the profitability premium.
3. If we hold everything in Equation 3 constant except for the expected growth in book value of equity, $dB_{t+\tau}$, and the expected stock return, $r$, then higher expected net asset growth must imply a lower expected stock return. All else equal, if two companies trade at the same relative price, the company with the higher investment must have a lower discount rate. This is an expression of the investment premium.

Measuring expected profitability and expected investment has been a challenge for many years. Novy-Marx (2013) documents the finding that profitability, measured by gross profits-to-assets, adds further explanatory power to asset pricing models. He found that controlling for gross profitability explains most earnings-related anomalies that the Three-Factor model had been unable to explain. Aharoni, Grundy, and Zeng (2013) document a weaker but statistically reliable inverse relationship between asset growth and average returns. Firms with aggressive investment policies, as measured by the growth in the book value of their assets, tend to have lower average returns.

Informed by the theoretical valuation equation and the advances in measuring expected profitability and investment, Fama and French (2015) propose a five-factor asset pricing model. The five factors include market beta, company size, relative price, gross profitability, and investment.

One of the most critical insights from the valuation equation is that the factors should not be considered in isolation. Empirically, Fama and French (1995) show that low relative price (value) stocks tend to have low profitability and investment, and growth stocks, particularly large growth stocks, tend to be profitable and invest aggressively. A portfolio focusing on profitability without controlling for relative price is likely to result in a portfolio of growth stocks. A portfolio focusing on relative price without controlling for profitability will likely result in a portfolio of stocks with weak profitability. Novy-Marx (2014) argues that buying stocks with robust profitability without paying premium prices is just as much value investing as buying average profitability assets at discount prices. The stocks with the highest expected returns in the market would be those with low relative prices and robust profitability. This makes targeting value and profitability jointly one of the most critical aspects of managing a multi-factor portfolio.

Company size was the original pricing anomaly. Interestingly, company size does not make an explicit appearance in the theoretical valuation equation, and the standalone size premium has not been statistically different from zero since the publication of the effect by Banz (1981). Based on this information, it would be easy to dismiss the inclusion of the size factor, but that would ignore one of the other empirical realities: other factor premiums are much stronger in small-cap stocks. Blitz and Hanauer (2021) empirically show powerful interaction effects between size and other factors, such as value. They show that academic factor portfolios, which consist of 50% large caps and 50% small caps, have significant alphas compared to factor portfolios constructed with 90% large caps and 10% small caps representing market capitalization weights. The conclusion is that the interaction between size and other known factors may be a sufficient reason for long-only investors to systematically overweight small-cap stocks, even if the size characteristic itself is not rewarded with a premium.

Theoretically, small companies may still fit (with a stretch) into the low-price phenomenon that explains the value premium. If current fundamentals are reasonable proxies for expected cash flows, low prices relative to fundamentals should be related to higher expected returns.

Given this research, we believe that the factors in the Fama and French Five-Factor model provide a good proxy for the priced risk factors available to investors; our factor-tilted expected return estimates reflect this by overweighting securities with exposure to these factors.

## Univariate Portfolio Sorts

To develop expectations for future factor premiums, we begin with history. The long-only portfolios are univariate sorts on company size, book-to-price equity, operating profitability, and investment. In some cases, we expect a premium from the "low" side of a sort, for example, a low size. In other cases, we expect it from the "high" side of the sort, for example high operating profitability. So, the tables have been arranged with the higher expected return side of the sort on the right.

Table 5 reports the sorted portfolios for size, book-to-price, profitability, and investment for US companies over the period July 1963 through May 2022 and for size and book-to-price over the period July 1926 through May 2022 based on the availability of data. For additional context, over the period from July 1963 through May 2022, the CRSP 1-10 index representing the capitalization-weighted US market returned 10.40% with a standard deviation of 15.32%, while over the period July 1926 through May 2022, it returned 10.06% with a standard deviation of 18.32%.

## Table 5 - US Stock Returns 7/1963 - 5/2022

| **Size** | Biggest 30% | Middle 40% | Smallest 30% |
|---|---|---|---|
| Annualized Return | 10.27% | 11.95% | 11.86% |
| Annualized Standard Deviation | 14.87% | 18.60% | 21.44% |

| **Book / Price** | Lowest 30% | Middle 40% | Highest 30% |
|---|---|---|---|
| Annualized Return | 10.21% | 10.81% | 13.45% |
| Annualized Standard Deviation | 16.17% | 15.01% | 17.37% |

| **Profitability** | Weakest 30% | Middle 40% | Most Robust 30% |
|---|---|---|---|
| Annualized Return | 7.99% | 10.35% | 11.83% |
| Annualized Standard Deviation | 18.16% | 15.08% | 15.23% |

| **Investment** | Aggressive 30% | Middle 40% | Conservative 30% |
|---|---|---|---|
| Annualized Return | 9.55% | 10.82% | 13.05% |
| Annualized Standard Deviation | 17.98% | 14.20% | 15.28% |

Source: PWL Capital; Data source: Ken French

## Table 6 - US Stock Returns 7/1926 - 5/2022

| **Size** | Biggest 30% | Middle 40% | Smallest 30% |
|---|---|---|---|
| Annualized Return | 9.97% | 11.65% | 11.80% |
| Annualized Standard Deviation | 17.89% | 23.09% | 28.65% |

| **Book / Price** | Lowest 30% | Middle 40% | Highest 30% |
|---|---|---|---|
| Annualized Return | 9.86% | 10.35% | 12.85% |
| Annualized Standard Deviation | 18.46% | 19.61% | 24.93% |

Source: PWL Capital; Data source: Ken French

Across all US sorts, we see a significant premium for the higher expected returning side over the lower expected returning side, and for the higher expected returning side over the market. Combining sorts, as predicted by the valuation equation, further increases expected (and historical) returns. An example of a multivariate sort would be the smallest stocks with high book-to-market and robust profitability. As predicted, this multivariate sort delivers a considerable premium with an annualized return from July 1963 through May 2022 of 17.57% with a standard deviation of 28.81%.

## Long-Short Factor Portfolios

Factor portfolios are constructed to proxy for sensitivity to common risk factors related to variables that capture the variation in returns. Factor portfolios are constructed as long-short portfolios, meaning that they own (long) the side of the factor that is expected to deliver a positive premium, like value stocks, while they are short the side of the factor that is expected to deliver a negative premium, like growth stocks. For the small company premium, we are observing the small company sort minus the big company sort (SMB); for the value premium, we are observing the high book-to-price sort minus the low book-to-price sort (HML); for the profitability premium, we are observing the robust minus the weak sort (RMW); and for the investment premium, we are observing the conservative sort minus the aggressive sort (CMA). Importantly, by construction, the Fama and French Five-Factor portfolios consist of 50% small stocks and 50% big stocks sorted on each variable. For example, HML (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios:

$$HML = 1/2\,(Small\,Value + Big\,Value) - 1/2\,(Small\,Growth + Big\,Growth)$$

Constructing the factor portfolios this way results in portfolios with similar weighted-average size, making the difference between the long and short returns essentially free of the size factor in returns. Table 7, Table 8, and Table 9 provide the historical factor returns for the US, developed ex-US and emerging markets. Using linear regression, we can measure an investment's sensitivity to the factors in the Fama and French Five-Factor model to approximate its sensitivity to the known drivers of expected returns. For example, if a fund has a regression loading of 1 on a factor, it would be expected to capture approximately 100% of that factor premium. Typically, long-only funds will have a loading below 1 to factors other than market beta.

### Table 7 - US Stock Premiums 7/1963 - 5/2022

| | MKT | SMB | HML | RMW | CMA |
|---|---|---|---|---|---|
| Annualized Return | 5.70% | 2.13% | 3.26% | 3.07% | 3.41% |
| Annualized Standard Deviation | 15.44% | 10.49% | 10.24% | 7.67% | 6.98% |

Source: PWL Capital; Data source: Ken French

### Table 8 - Developed Markets ex-US Premiums 7/1992 - 5/2022

| | MKT | SMB | HML | RMW | CMA |
|---|---|---|---|---|---|
| Annualized Return | 3.79% | 0.73% | 3.86% | 3.61% | 1.52% |
| Annualized Standard Deviation | 15.97% | 6.75% | 8.18% | 4.74% | 6.31% |

Source: PWL Capital; Data source: Ken French

### Table 9 - Emerging Markets Premiums 7/1992 - 5/2022

| | MKT | SMB | HML | RMW | CMA |
|---|---|---|---|---|---|
| Annualized Return | 5.01% | 0.90% | 7.69% | 2.09% | 3.08% |
| Annualized Standard Deviation | 20.95% | 7.24% | 7.99% | 5.55% | 6.66% |

Source: PWL Capital; Data source: Ken French

## Historical Factor Premiums

There is a tremendous amount of variability in the factor premiums across time and regions. We estimate the historical premiums for the world as the market-cap weighted average of the developed and emerging markets. Table 10 displays the historical world premiums. Next, we apply a shrinkage factor to the historical premiums discussed in the following section to estimate the forward-looking premiums.

### Table 10 - World Historical Premiums 7/1992 - 5/2022

| | MKT | SMB | HML | RMW | CMA |
|---|---|---|---|---|---|
| Annualized Return | 5.54% | 0.36% | 3.07% | 3.91% | 2.65% |
| Annualized Standard Deviation | 15.08% | 6.27% | 8.69% | 4.88% | 6.53% |

Source: PWL Capital; Data source: Ken French

## The Decline of Out-of-Sample Alpha

### *Pre/Post Publication Alpha*

The decline in post-publication alpha for each non-market risk factor was investigated for the US, developed markets ex-US, and emerging markets using Ken French's 5-factor data up to and including April 2022. The full results are presented in Table 11. For this analysis, the 1993 publication date of the Fama-French 3-factor model was considered the in/out of sample breakpoint for the size and value factors, while the 2015 publication date of the Fama-French 5-factor model was considered for the profitability and investment factors. Despite the individual factors being found earlier than these dates, the publication dates of the Fama-French papers more accurately represent when the implementation of the factors into portfolio management strategies began.

## Table 11 - Pre/Post-Publication Alphas for the Four Non-Market Risk Factors of the Fama-French 5-Factor Model for the US, Developed ex-US and Emerging Markets

| | Full Data | Pre Pub | Post Pub | PP Decline |
|---|---|---|---|---|
| **US** | | | | |
| SMB | 0.00110 | 0.00220 | 0.00010 | 95.5% |
| HML | 0.00380 | 0.00510 | 0.00230 | 54.9% |
| RMW | 0.00330 | 0.00330 | 0.00300 | 9.1% |
| CMA | 0.00390 | 0.00420 | 0.00190 | 54.8% |
| Average | 0.00303 | 0.00370 | 0.00183 | 50.7% |
| **Developed Ex-US** | | | | |
| SMB | 0.00120 | 0.00000 | 0.00150 | - |
| HML | 0.00330 | 0.00000 | 0.00360 | - |
| RMW | 0.00370 | 0.00390 | 0.00330 | 15.4% |
| CMA | 0.00170 | 0.00260 | 0.00000 | 100.0% |
| Average | 0.00248 | 0.00163 | 0.00210 | -29.2% |
| **Emerging** | | | | |
| SMB | 0.00260 | 0.01260 | 0.00140 | 88.9% |
| HML | 0.00630 | 0.00260 | 0.00670 | -157.7% |
| RMW | 0.00260 | 0.00260 | 0.00260 | 0.0% |
| CMA | 0.00330 | 0.00390 | 0.00170 | 56.4% |
| Average | 0.00370 | 0.00543 | 0.00310 | 42.9% |

Sources: Mclean and Pontiff; Jensen, Kelly and Pederson

The decline in post-publication alpha in US and Emerging markets agrees with the findings of McLean and Pontiff (2015) and Jensen, Kelly, and Pedersen (2021) of 58% and 47%, respectively. However, due to the lack of data for Developed ex US markets, the Developed ex US data showed a different relationship, with a larger alpha post-publication. The time series beginning in July 1990 left only ~2.5 years of in-sample data – insufficient to draw conclusions. The same can be said for the Emerging markets data set, with only ~3.5 years of in-sample data pre-1993. As a result, we consider the 50.7% decline in US post-publication alpha to be the only meaningful result out of the three regions studied.

## *Bayesian Shrinkage Factor*

The Bayesian approach introduced by Jensen, Kelly and Pedersen (2021) was applied to this work to determine the Bayesian shrinkage factor, SF, to be applied to the posterior alpha. As the authors derived, the shrinkage factor can be calculated using Eq. 4.

$$SF = \frac{1}{1+\frac{\sigma^2}{\tau^2 T}} \tag{4}$$

where $\tau^2$ is the variance around a zero-mean alpha, $\sigma^2$ is the variance of the OLS error terms, and $T$ is the sample size. The shrinkage factor then is multiplied by the historical alpha, $\hat{\alpha}$, to generate the posterior (future expected) alpha, $E(\alpha|\hat{\alpha})$, illustrated by Eq. 5. Since the shrinkage factor is bound between 0 and 1, the posterior alpha will be bound between 0 and the historical alpha.

$$E(\alpha|\hat{\alpha}) = SF * \hat{\alpha} \tag{5}$$

To examine the consequences of Eq. 4, let us examine its behavior at the extremes for each of the three variables, $\sigma$, $\tau$, and $T$, while holding the other variables constant.

For example, for the case of $\sigma$ = 0 (meaning 0% standard deviation of error terms – or a high confidence level in the model), SF would simplify to 1/(1+0) = 1. In this case, the expected future alpha would equal 100% of the historical mean alpha. That is, we would expect the same alpha moving forward as we have seen in the past, again because we have high confidence in the model.

In the reverse case of high σ (high standard deviation of error terms – low confidence in the model), the shrinkage factor would trend towards zero, meaning that the future expected alpha would trend towards zero as well (see Eq. 5). Since we have low confidence in the model, we cannot rule out that a significant portion of historical alpha was due to luck, hence a lower expected alpha going forward.

Suppose the standard deviation around a zero-mean alpha, $\tau$, approaches zero. In that case, this means we have high confidence that the true alpha is zero, or in other words, the deviations from the CAPM model are zero. That is, the non-market risk factors do not generate outperformance, and the market can capture the entire performance of the factor. So, looking at Eq. 4, when $\tau^2 \rightarrow 0$, the shrinkage factor also goes to zero. This would lead the expected future alpha to be zero (Eq. 5), which would make sense given that we firmly believe the factor does not generate alpha.

The shrinkage factor will approach one when we have a high standard deviation around a zero-mean alpha. The high standard deviation around a zero-mean alpha tells us that the deviations from the CAPM model are high, and we have low confidence that the alpha is zero. The higher the deviations from CAPM, the higher the shrinkage factor, and the closer the expected alpha will match the historical alpha.

Finally, looking at the number of samples, $T$, we observe the simple relationship that with more data (higher $T$), the higher the shrinkage factor, the more confident we are that the historical alpha will persist (high expected alpha). With a lower amount of data (low $T$), the lower the shrinkage factor, the less confident we are that the alpha will persist (low expected alpha).

To begin the analysis, a simple ordinary least squares (OLS) regression was performed on each non-market risk factor against the market. The OLS regression is governed by Eq. 6

$$r_t^f = \alpha + \beta r_t^m + \varepsilon_t \tag{6}$$

where $r_t^f$ is the return of the non-market risk factor at time $t$, $r_t^m$ is the return of the market at time and $t$, $\alpha$, and $\beta$ are the OLS regression parameters for intercept and slope, respectively, and $\varepsilon_t$ is the error term. The OLS analysis was performed for the four non-market risk factors for the three regions studied. The average shrinkage factor across the three regions was 0.91, in agreement with the 0.9 shrinkage factor calculated by Jensen, Kelly and Pedersen (2021). Table 12 summarizes the shrinkage factor for each of the three regions.

## Table 12 - Average Shrinkage Factor for the US, Developed ex-US, and Emerging Markets

| | SF |
|---|---|
| US | 0.94 |
| Developed Ex-US | 0.87 |
| Emerging Markets | 0.91 |
| Average | 0.91 |

Source: PWL Capital; Data source: Ken French

### *Expected Alpha*

The results from the Pre/Post Publication Alpha suggest a 0.5 shrinkage factor in post-publication alpha, while the results from the Bayesian approach suggest a 0.9 shrinkage factor. To calculate the expected alpha, we have used the average result between the two approaches, resulting in a 0.7 shrinkage factor, which is then applied to the historical alpha to generate the expected alpha using Eq. 5. Table 13 summarizes our expected alphas for each factor.

## Table 13 - Expected Alpha for the US, Developed ex-US, and Emerging Markets

| | US | | Developed Ex-US | | Emerging | |
|---|---|---|---|---|---|---|
| | $\hat{\alpha}$ | $E(\alpha\|\hat{\alpha})$ | $\hat{\alpha}$ | $E(\alpha\|\hat{\alpha})$ | $\hat{\alpha}$ | $E(\alpha\|\hat{\alpha})$ |
| SMB | 0.11% | 0.08% | 0.12% | 0.08% | 0.26% | 0.18% |
| HML | 0.38% | 0.27% | 0.33% | 0.23% | 0.63% | 0.44% |
| RMW | 0.33% | 0.23% | 0.37% | 0.26% | 0.26% | 0.18% |
| CMA | 0.39% | 0.27% | 0.17% | 0.12% | 0.33% | 0.23% |

Source: PWL Capital; Data source: Ken French

# Forward-Looking Factor Premium: Putting it All Together

Table 14 displays the world historical premiums (reproduced from Table 10) for the SMB (size), HML (relative price), RMW (operating profitability) and CMA (investment). Those historical premiums are adjusted downward by a shrinkage factor of 0.7 to reflect the tendency of premiums to decline following their publication in the scientific literature, thus producing the expected world premiums.

## Table 14 - World Premiums 7/1992 - 5/2022

| | SMB | HML | RMW | CMA |
|---|---|---|---|---|
| Historical Premiums | 0.36% | 3.07% | 3.91% | 2.65% |
| Expected Premiums (Shrinkage factor = 0.7) | 0.25% | 2.15% | 2.74% | 1.86% |

Source: PWL Capital; Data source: Ken French

Next, we multiply the individual source funds' factor loadings by the world expected factor premiums to arrive at a final factor premium for the factor-tilted portfolio. To evaluate these loadings, we have studied the period 1/2012 to 11/2022 for the component funds of the DFA Global Equity Portfolio. The factor premiums for the component funds and the DFA Global Equity Portfolio are displayed in the last column of Table 15.

## Table 15 - Factor Analysis of DFA Canada Equity Funds 1/2012 – 11/2022

| | | 5 FACTORS | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Market | Size | Relative price | Profitability | Investment | | |
| | Alpha | RM-RF | Small - Big | Value - Growth | Robust - Weak | Conservative-Aggressive | R2 | Gross Expected Premium |
| Canadian Core | | | | | | | | |
| Factor loading | 0.04 | 1.01 | 0.10 | 0.17 | 0.01 | -0.02 | 0.994 | 0.39% |
| t stat | 1.07 | 76.28 | 5.37 | 12.97 | 0.40 | -1.42 | | |
| Canadian Vector | | | | | | | | |
| Factor loading | 0.01 | 1.03 | 0.27 | 0.27 | -0.03 | -0.05 | 0.990 | 0.54% |
| t stat | 0.16 | 51.76 | 9.53 | 13.75 | -1.42 | -2.09 | | |
| U.S. Core | | | | | | | | |
| Factor loading | 0.01 | 0.99 | 0.10 | 0.13 | 0.08 | 0.03 | 0.997 | 0.53% |
| t stat | 0.26 | 126.45 | 7.21 | 10.61 | 4.59 | 1.46 | | |
| U.S. Vector | | | | | | | | |
| Factor loading | -0.03 | 1.01 | 0.28 | 0.37 | 0.01 | -0.02 | 0.994 | 0.86% |
| t stat | -0.69 | 81.72 | 12.15 | 18.47 | 0.29 | -0.63 | | |
| International Core | | | | | | | | |
| Factor loading | 0.00 | 1.06 | 0.01 | 0.16 | 0.17 | -0.03 | 0.992 | 0.81% |
| t stat | -0.01 | 72.43 | 0.23 | 3.08 | 2.43 | -0.40 | | |
| International Vector | | | | | | | | |
| Factor loading | -0.02 | 1.08 | 0.13 | 0.31 | 0.18 | -0.10 | 0.992 | 1.20% |
| t stat | -0.30 | 70.26 | 2.86 | 5.72 | 2.46 | -1.17 | | |
| Global Portfolio | | | | | | | | 0.64% |

Source: PWL Capital; Data source: Ken French, DFA

The last column of Table 15 displays each fund's expected premiums. We also know that the DFA Global Equity Fund comprises 70% Core funds and 30% Vector funds. The weights of Canadian, US and international equity within the DFA Global Equity Fund are also known (34%/41%/25%). We can then calculate the gross expected premium for the DFA Global Equity Fund (0.64%).

As a final step, we subtract the MER estimate of the DFA Global Equity Fund in the coming year to obtain the net expected premium figure:

## Table 16 - Global Equity Fund Net Factor Premium

| Gross Expected Premium | Minus: Estimated MER | Equals: Net Expected Premium |
|---|---|---|
| 0.64% | 0.34% | 0.30% |

Source: PWL Capital; Data source: Ken French, DFA

# 4. Expected Inflation

Inflation significantly impacts real returns and is a concern for investors. In previous versions of the FPA methodology, we have used the breakeven yield of Real Return Bonds (RRBs) issued by the federal government as a market-based measure of long-term expected inflation. While there is little doubt (based on logic) that RRB breakeven yields, to some extent, reflect inflation expectations, our data review failed to confirm a high predictive power. In addition, the Canadian federal government stopped issuing RRBs in late 2022; therefore, we are concerned that the quoted prices on these securities will be backed by too little trading volume to reflect the market's inflation expectations properly. From now on, we will estimate expected inflation from the average of only two indicators: the Bank of Canada's target inflation rate and the historical rate since 1900.

### Table 17 - Expected Inflation Composition

| | 0.5 x (Historical Inflation) Plus | 0.5 x (Target Inflation) | Equals Expected Inflation |
|---|---|---|---|
| Inflation | 3.0% | 2.0% | 2.4% |

Source: PWL Capital; Data Sources: Elroy Dimson, Paul Marsh and Mike Staunton, Triumph of Optimists: 101 Years of Global Investment Returns, Princeton University Press, 2002; Elroy Dimson, Paul Marsh and Mike Staunton, Credit Suisse Global Returns Yearbook and Sourcebook, 2018, Zurich: Credit Suisse Research Institute, 2021, Bank of Canada

# 5. Primary Residence

## Expected Price Appreciation

Distinct from stocks and bonds, the primary residence is often one of the most significant assets owned by a household. Understanding the expected returns for housing is essential in financial planning. The primary residence is not always an asset that will be sold to fund consumption (though it is always a possibility). Still, the total housing costs are related to the asset's expected return. Just as a renter pays rent, an owner similarly has unrecoverable housing costs consisting of property taxes, maintenance costs, and the cost of capital. When real estate expected returns are lower than the stock market expected returns, home equity comes with an opportunity cost, as the home equity could be invested in stocks. The concept of the User Cost of housing described by Himmelberg *et al.* (2005) includes the opportunity cost of capital, property taxes, maintenance costs, and a risk premium for the additional risk of owning rather than renting. Theoretically, this total cost should equate to the cost of renting a place to live, though that will often not be true practically. In some markets, renting may be more attractive than owning, and the opposite will be true in other markets. To understand the User Cost of housing, we need to document the opportunity cost and the expected capital appreciation from real estate.

Based on global data dating as far back as 1628, Shiller (2006) estimates long-term historical real home price appreciation at between 0.2% and 0.4% per annum. Jorda *et al.* (2017) estimate the real annual historical increase of home prices in 16 countries from 1870 to 2015 at 1.1%. Importantly, this is a global figure, while no homeowner owns global real estate. Many of the countries in the sample had a lower appreciation, while others were a higher one. The wide dispersion in returns stemming from concentration in a single country, city, and asset is one of the risks of owning real estate.

In many cities in the US and Canada, this risk has paid off handsomely in recent years. From 2001 to 2020, the S&P CoreLogic Case-Shiller U.S. National Home Price Index grew by a real annualized rate of 1.8%, while the Teranet-National Bank Canadian Home Price Index increased by an annualized rate of 4.7%. These indices do not account for maintenance costs. Additionally, period-specific factors have buoyed price appreciation and may not repeat, such as the decline of five-year government bond yields from 5.1% to 0.9% in the U.S. and from 5.3% to 0.7% in Canada.

These capital appreciation figures do not include maintenance costs estimated at between 1% and 2% historically by Jorda *et al.* (2017); their estimate includes depreciation and all other housing-related expenses excluding interest, taxes, and utilities. Statistics Canada uses 1.5% of the home's value as a depreciation expense in the CPI basket; this figure aligns with multiple academic studies and the statistical agencies of other countries. 1.5% is for the building only, so it must be multiplied by the ratio of the building over the land value to arrive at the depreciation cost for the entire home's value estimate expected real capital return for personal residences at 1%. The carrying costs, including maintenance, insurance, and property taxes must also be captured. We estimate a 1% annual cost for maintenance and insurance. As property taxes vary greatly, we do not attempt to prescribe a figure here, but users should be sure to include them based on their circumstances. A 1% real return, less maintenance and property taxes (not to mention the opportunity cost of home equity) may make housing look like a poor investment. Still, it is essential to remember that the owner receives imputed rent as a benefit. The User Cost of housing would suggest that the total costs of an owner and renter should be similar.

Homeownership is consumption as much as an investment, though the Canadian experience of rising real estate prices makes this easy to forget. One unique consideration in assessing the risk and return of home ownership is that an owned home provides a perfect hedge against the cost of consuming that specific home, as described in Barras and Betermier (2020). However, similar to a long-term bond, the short-term price volatility of this hedge can make it a risky investment for a short-term owner. Due to the heterogeneity of real estate markets, we do not attempt a predictive approach to determining expected real estate appreciation. It should be noted that Himmelberg *et al.* (2005) and Case and Shiller (2004) suggest that a housing bubble occurs when homebuyers are willing to pay inflated prices for houses today because they expect unrealistically high housing appreciation in the future. If the User Cost of owning property in a market exceeds renting, prices may be too high and expected returns may be lower than average.

In financial planning, we typically capture the appreciation on the real estate asset and model maintenance and other carrying costs as cash flow requirements while the home is owned.

# Expected Volatility

We estimate the expected volatility of individual primary residences as the sum of the market volatility and the idiosyncratic volatility.

The market volatility of Canadian homes is estimated using the average of the 5- and 20-year standard deviation of the Teranet/National Bank C11 Index, which currently stands at 3.5%.

To estimate the idiosyncratic return volatility of residences, we found two studies documenting the total volatility of individual homes in the US. Our first step was determining whether US data could provide sound evidence in a Canadian context. We looked at the S&P Case/Shiller and the Teranet/National Bank indices for insight. Although these two indices are only moderately correlated (0.55 from March 1999 to November

2022), their volatilities are similar (3.5% per annum for the Canadian Index compared to 3.0% for the US). Based on this evidence, we assume the idiosyncratic volatility of Canadian and US homes will be similar.

Haurin & Zhou (2010) document the volatility of US individual homes from 1985 to 2003, and Peng & Thibodeau (2016) cover the periods from 1996 to 2000, 2001 to 2007, and 2007 to 2012. We calculate an average idiosyncratic volatility of 10.6% from these studies and add it to the general Canadian market volatility to obtain an estimate of 14.1% for the total volatility of Canadian homes, as documented in Table 18 below.

### Table 18 - Canadian Individual Home Volatility Estimate

| | | |
|---|---|---|
| Canadian Market Volatility Estimate (3/1999-12/2022) | | 3.5% |
| Plus: Idiosyncratic Volatility: | | |
| Haurin & Zhou (1985-2003) | 13.7% | |
| Peng & Thibodeau (1996-2000)[3] | 9.4% | |
| Peng & Thibodeau (2001-2007) | 7.9% | |
| Peng & Thibodeau (2007-2012) | 11.5% | |
| Average | 10.6% | 10.6% |
| **Total Volatility** | | **14.1%** |

Source: PWL Capital; Data Sources: Haurin and Zhou, Peng and Thibodeau, Federal Reserve Bank of Saint-Louis, Teranet/National Bank

# 6. Expected Cost of Borrowing

Borrowing money is equivalent to taking a short position in a fixed-income security. Therefore, the methodology for evaluating the expected cost of borrowing will be similar to that of a bond. Like we do for asset class expected returns, we estimate the cost of debt with a combination of the MBER and ECOC for each type of loan. We estimate the cost of borrowing for three different instruments:

- The cost of a five-year fixed-rate mortgage is analogous to the expected return on a five-year bond.
- The cost of secured and unsecured lines of credit ("LOC") is similar the expected return of two floating-rate bonds of different creditworthiness.

We estimate the MBER for each instrument with the data series "Funds advanced, outstanding balances, and interest rates for new and existing lending" from the Bank of Canada. Unfortunately, this series, which documents interest rates granted on actual bank loans, is published with a two-month lag. We make up for this staleness by adjusting the two-month-old rate with the posted (mortgage and prime) rates published by the Bank of Canada, which is always up to date. For example, if in the two-month interval the prime rate increased, we augment the observed LOC rate by the amount of this increase. As of April 30, 2023, the weighted average interest rate on five-year mortgages was 5.02% and the posted rate remained unchanged at the end of June, for a MBER of 5.02%. The weighted average interest rates on secured and unsecured lines of credit were 6.57% and 10.16% at the end of April 2023 and the prime rate increased by 0.25% during the May-June period, so the MBERs at the end of June were 6.82% and 10.41% respectively.

[3] Haurin & Zhou provide an estimate for the total volatility of US homes (15%) from which we subtract the volatility of the S&P Case/Shiller Index for 1987-2003 (1.3%).

The ECOC for 5-year fixed rate mortgages is based on the ECOC of the FTSE Canada Short Term Bond Index plus a historical spread. As of the end of June 2023, the ECOC for the short bond index is estimated at 3.49% plus an average historical spread of 1.31% from 2013 to mid-year 2023, for a total ECOC for five-year mortgages of 4.80%. We estimate the ECOC for LOC borrowings based on the ECOC of cash instruments plus a historical spread over 3-month Treasury Bill returns. The ECOC for cash at the end of June 2023 is estimated to 2.95% plus an average spread of 2.40% for secured LOCs and 5.43% for unsecured LOCs over the period 2013 to mid-year 2023, resulting in ECOCs of 5.35% and 8.38% respectively.

As a final step, we apply the appropriate weights W1 and W2 to the MBER and ECOC to obtain the expected cost of borrowing of each type of loan. We use the W1 weighting of fixed-income (75%) for 5-year mortgages and the W1 of cash instruments (15%) for lines of credit, as outlined in Table 19.

### Table 19 - Cost of Borrowing

| | W1 | Nominal MBER | W2 | Nominal ECOC | Expected Cost of Borrowing |
|---|---|---|---|---|---|
| Five-Year Mortgage - Fixed rate | 75% | 5.02% | 25% | 4.80% | 4.97% |
| Personal Line of Credit - Secured | 15% | 6.82% | 85% | 5.35% | 5.57% |
| Personal Line of Credit - Unsecured | 15% | 10.41% | 85% | 8.38% | 8.68% |

Source: PWL Capital; Data Sources: Statistics Canada, Bank of Canada, Bloomberg, BMO Global Asset Management. Data as of June 30, 2023.

# 7. Investment Portfolios: Underlying Indices

When evaluating the market-based expected return, standard deviation, and correlation of asset classes for the market-cap-weighted portfolios, we rely on the market indices described in Table 20.

### Table 20 - Market-Cap-Weighted Portfolio Indices

| Asset Class | Market Index |
|---|---|
| Fixed Income | Canada Bond Universe Index |
| Canadian Equity | Canada Total Market Index |
| US Equity | US Total Market Index |
| International Equity | International Developed and Emerging Market Index |

Source: PWL Capital

World factor premiums and DFA funds' factor loadings are calculated using the Ken French Data Library and DFA Returns Web. Dimensional Fund Advisors provide Canadian equity factor data. The ECOCs for all asset classes were estimated using the Dimson, Marsh, and Staunton database. Standard deviations and asset class correlations for factor-tilted portfolios are estimated from the market indices described in Table 22.

### Table 21 - Factor-Tilted Portfolio Indices

| Asset Class | Market Index |
|---|---|
| Fixed Income - Factor-Tilted | 40% Canada Short-Term Bond Index +<br>25% Canada Short-Term Corporate Bond Index +<br>35% Canada Universe Bond Index |
| Canadian Equity - Factor Tilted | 70% DFA Canadian Core Index +<br>30% DFA Canadian Vector Index |
| US Equity - Factor Tilted | 70% DFA US Core Index +<br>30% DFA US Vector Index |
| International Equity - Factor Tilted | 70% DFA International Core Index +<br>30% DFA International Vector Index |

Source: PWL Capital

# 8. Investment Portfolios: Underlying Funds

In order to calculate expected returns (net of fees) and the composition of asset class returns, we identify funds that provide appropriate fee and distribution yield estimates. Table 22 designates the ETFs used for market-cap weighted portfolios, while Table 23 designates the DFA mutual funds used for factor-tilted portfolios.

### Table 22 - Market-Cap Weighted Portfolio Representative ETFs

| Asset Class | ETF (Ticker) |
|---|---|
| Fixed Income | Vanguard Aggregate Bond (VAB) |
| Canadian Equity | BMO S&P/TSX Capped Composite Index ETF (ZCN) |
| US Equity | Vanguard U.S. Total Market Index ETF (VUN) |
| International Equity | 70% Vanguard FTSE Developed All Cap ex North America Index ETF (VIU) +<br>30% Vanguard FTSE Emerging Markets All Cap Index ETF (VEE) |

Source: PWL Capital

### Table 23 - Factor-Tilted Portfolio Representative Mutual Funds

| Asset Class | Mutual Fund (Fundserv code) |
|---|---|
| Fixed Income - Factor-Tilted | DFA Global Fixed Income – F Class (DFA 916) |
| Canadian Equity - Factor Tilted | 70% DFA Canadian Core Index – F Class (DFA 256) +<br>30% DFA Canadian Vector Index – F Class (DFA 600) |
| US Equity - Factor Tilted | 70% DFA US Core Index – F Class (DFA 293) +<br>30% DFA US Vector Index – F Class (DFA 223) |
| International Equity - Factor Tilted | 70% DFA International Core Index – F Class (DFA 295) +<br>30% DFA International Vector Index – F Class (DFA 227) |

Source: PWL Capital

# 9. Expected Returns – Market-Cap Weighted Portfolios

We estimate asset class expected returns with a weighted average of the Market-Based Expected Return (MBER) and the Equilibrium Cost of Capital (ECOC). The MBER is an estimate of expected returns based on current market conditions. The MBER is based on the yield to maturity for fixed income and 1/CAPE for equity. The ECOC estimates expected returns based on more than 120 years of global asset class return historical data – adjusted for non-recurring items. As explained in Section 2, the weighting of each component is derived from the statistical explanatory power of the MBER. Empirical evidence suggests that the MBER has a high explanatory power for fixed income and a relatively low (but significant) explanatory power for equity and cash instruments.

We attribute a weight “W1” to the MBER and the balance of the attribution “W2” to the ECOC to obtain gross asset class returns. W1 and W2 are estimated to 15%/85% for cash instruments, 75%/25% for fixed-income, and 25%/75% for stocks. We then subtract the ETF MERs to get the net nominal expected return, as outline in Table 24.

### Table 24 - Market-Cap Weighted Asset Class Expected Returns

| Asset Class | W1 | Nominal MBER | W2 | Nominal ECOC | Gross Nominal Expected Return | Minus: MER | Net Expected Nominal Return |
|---|---|---|---|---|---|---|---|
| Cash | 15% | 4.26% | 85% | 3.07% | 3.25% | 0.00% | 3.25% |
| Short Term Fixed Income | 75% | 4.40% | 25% | 3.61% | 4.20% | 0.11% | 4.09% |
| Fixed Income | 75% | 4.28% | 25% | 4.15% | 4.25% | 0.09% | 4.15% |
| Canadian Equity | 25% | 6.77% | 75% | 7.08% | 7.00% | 0.05% | 6.95% |
| US Equity | 25% | 5.64% | 75% | 7.08% | 6.72% | 0.15% | 6.56% |
| International Equity (DV+EM) | 25% | 9.42% | 75% | 7.08% | 7.66% | 0.23% | 7.41% |
| Global Equity | | | | | 7.05% | 0.14% | 6.91% |

Source: PWL Capital; Data Sources: Bloomberg, DFA Returns Web, Robert Shiller, Elroy Dimson, Paul Marsh and Mike Staunton, Triumph of the Optimists: 101 Years of Global Investment Returns, Princeton University Press, 2002; Elroy Dimson, Paul Marsh and Mike Staunton, Credit Suisse Global Returns Yearbook and Sourcebook, 2018, Zurich: Credit Suisse Research Institute, 2021

# 10. Expected Return – Factor-Tilted Portfolios

## Fixed Income

The DFA Global Fixed Income Portfolio is a fund-of-fund made of three components: the DFA Five-Year Global Fixed Income Fund, the DFA Global Targeted Credit Fund, and the DFA Global Investment Grade Fixed Income Fund. Table 25 outlines the weightings of all component funds and the proxy indices in use.

### Table 25 - Structure of the Factor-Tilted Fixed-Income Portfolio

| Fund | Fundserv Code | Proxy Index | Weight |
|---|---|---|---|
| DFA Five-Year Global Fixed Income Fund (F) | DFA231 | Canada Short Bond Index | 40% |
| DFA Global Targeted Credit Fund (F) | DFA857 | Canada Short Corporate Bond Index | 25% |
| DFA Global Investment Grade Fixed Income Fund (F) | DFA449 | Canada Universe Bond Index | 35% |
| DFA Global Fixed Income Portfolio (F) | DFA916 | Factor-Tilted Fixed-Income portfolio | 100% |

Source: PWL Capital; Data Source: DFA

## *Market-Based Expected Return*

The MBER for factor-tilted fixed income will be a weighted average of the yield-to-maturity of the proxy market indices that mimic the underlying funds to the DFA Global Fixed Income Portfolio. The details are outlined in Table 26. By our calculations, the gross MBER of the factor-tilted fixed-income portfolio equals 4.58%.

### Table 26 - Nominal Gross MBER of the Factor-Tilted Fixed-Income Portfolio

| Proxy Index | Weight | Yield-to-maturity |
|---|---|---|
| (as of December 31, 2022) | DFA231 | 40% |
| Canada Short Bond Index | 40% | 4.40% |
| Canada Short Corporate Bond Index | 25% | 5.28% |
| Canada Universe Bond Index | 35% | 4.28% |
| **Factor-Tilted Fixed-Income Portfolio Gross MBER** | | **4.58%** |

Source: PWL Capital; Data Source: BMO, DFA

## *Equilibrium Cost of Capital*

The base for estimating the ECOC of the factor-tilted fixed-income portfolio is the Canada Bond Universe Index, which we adjust for the difference in duration and credit exposure of the component funds. The process is detailed in Table 28.

We estimate the ECOC of the Canada Universe Index with the DMS historical real return for global bonds in US dollars from 1901 to 2022, which equals 1.75%.

The discount for the shorter maturities of the DFA231 and the DFA857 compared to the Canada Universe Index is estimated from the average yield difference between the Canada Short-Term Bond Index and the Canada Universe Bond Index from December 1985 to December 2022. This so-called "maturity discount" equals -0.55%.

The premium to account for the greater exposure of the DFA857 to credit risk compared to the Canada Universe Index is estimated from the average yield difference between the Canada Short-Term Corporate Bond Index and the Canada Short-Term Bond Index from December 1985 to December 2022. This so-called "credit premium" equals 0.58%.

The weighted average of the component funds results in a real ECOC estimate of 1.53%. Once adding our expected inflation estimate (2.40%), we obtain a nominal gross ECOC estimate of 3.94%.

## Table 27 - Nominal Gross ECOC of the Factor-Tilted Fixed-Income Portfolio

| Underlying fund | Index | Weight | Bond Universe Real ECOC | Minus: maturity discount | Plus: credit premium | Total |
|---|---|---|---|---|---|---|
| DFA 231 | FTSE Canada Short Bind Index | 40% | 1.75% | -0.55% | NA | 1.20% |
| DFA 857 | FTSE Canada Short Corporate Bond Index | 25% | 1.75% | -0.55% | 0.58% | 1.78% |
| DFA 449 | FTSE Canada Universe Bond Index | 35% | 1.75% | NA | NA | 1.75% |
| | | | | | | |
| Real ECOC | | | | | | 1.53% |
| Plus: | Expected Inflation | | | | | 2.40% |
| **Factor-Tilted Fixed-Income Portfolio Gross ECOC** | | | | | | **3.94%** |

Source: PWL Capital; Data Source: Dimson, Marsh and Staunton, BMO, Bloomberg

### *Expected Return*

As described in Table 28, for fixed-income securities, we attribute a weight of 75% to the MBER and 25% to the ECOC, which results in a gross expected return for the factor-tilted portfolio of 4.42%, from which we subtract 0.31% for the MER of the DFA Global Fixed Income Portfolio (F class) to obtain a net expected return for the factor-tilted fixed-income portfolio of 4.09%.

## Table 28 - Expected Return of the Factor-Tilted Fixed Income Portfolio

| | Weight | Gross Expected Return | MER | Net Expected Return |
|---|---|---|---|---|
| MBER | 75% | 4.58% | | |
| ECOC | 25% | 3.94% | | |
| Expected Return | | 4.42% | 0.31% | 4.09% |

Source: PWL Capital; Data Source: Dimson, Marsh and Staunton, BMO, Bloomberg

# Equity

Estimating the factor-tiled equity expected returns involves several steps. First, we calculate the gross factor premium for Canadian, US, international, and global markets. Table 15 provides the gross premium estimates for the relevant DFA equity funds. Since the Core and Vector funds weigh 70% and 30% respectively in the DFA Global Equity fund, we can calculate the gross premium for Canadian, US and international equity. Next, we calculate the gross premium for global equity from the regional weights and regional premiums. The gross factor premiums are 0.43%, 0.63%, 0.93%, and 0.64% respectively, as detailed in Table 29.

## Table 29 - Gross Regional Premiums

| Regional Weight | | Gross Core Premium | Core Weight | Gross Vector Premium | Vector Weight | Gross Factor Premium |
|---|---|---|---|---|---|---|
| 33% | Canadian Equity | 0.39% | 70% | 0.54% | 30% | 0.43% |
| 41% | US Equity | 0.53% | 70% | 0.86% | 30% | 0.63% |
| 26% | International DV & EM Equity | 0.81% | 70% | 1.20% | 30% | 0.93% |
| 100% | Global Equity | | | | | 0.64% |

Source: PWL Capital; Data source: DFA, Ken French, Dimson, Marsh and Staunton

Finally, we add the gross factor premiums to the market-cap weighted gross expected returns (Ref: Table 24) and we subtract the MER for each asset class to obtain the net factor-tilted expected returns. As outlined in Table 30, the net factor-tilted expected returns for Canadian, US, international, and global equity are 7.16%, 7.10%, 8.17% and 7.37%.

## Table 30 - Asset Class Expected Returns of the Factor-Tilted Equity Portfolio

| | Market-Cap-Weighted Expected Return (Gross) | Plus: Gross Factor Premium | Factor-Tilted Expected Return (Gross) | Minus: MER | Factor-Tilted Expected Return (Net) |
|---|---|---|---|---|---|
| Canadian Equity | 7.00% | 0.43% | 7.46% | 0.28% | 7.16% |
| US Equity | 6.72% | 0.63% | 7.39% | 0.27% | 7.10% |
| International Equity DV & EM | 7.66% | 0.93% | 8.67% | 0.45% | 8.17% |
| Global Equity - Factor-Tilted | 7.05% | 0.64% | 7.74% | 0.34% | 7.37% |

Source: PWL Capital; Data Sources: Bloomberg, DFA Returns Web, Robert Shiller, Elroy Dimson, Paul Marsh and Mike Staunton, Triumph of the Optimists: 101 Years of Global Investment Returns, Princeton University Press, 2002; Elroy Dimson, Paul Marsh and Mike Staunton, Credit Suisse Global Returns Yearbook and Sourcebook, 2018, Zurich: Credit Suisse Research Institute, 2021.

## Expected Returns of Factor-Tilted Portfolios – All Asset Classes

The expected returns for all factor-tilted assets classes are summarized in Table 31 below.

### Table 31 – Expected Returns of Factor-Tilted Asset Classes

| Asset Class | W1* | Nominal MBER | W2* | Nominal ECOC | Nominal Expected Return - Gross of fees | MER | Nominal Expected Return - Net of fees |
|---|---|---|---|---|---|---|---|
| Cash | 75% | 4.26% | 25% | 3.07% | 3.96% | 0.00% | 3.96% |
| Fixed Income - Factor-Tilted | 75% | 4.58% | 25% | 3.94% | 4.42% | 0.31% | 4.09% |
| Canadian Equity - Factor-Tilted | 25% | 7.23% | 75% | 7.54% | 7.46% | 0.28% | 7.16% |
| US Equity - Factor-Tilted | 25% | 6.31% | 75% | 7.75% | 7.39% | 0.27% | 7.10% |
| International Equity DV & EM - Factor-Tilted | 25% | 10.44% | 75% | 8.07% | 8.67% | 0.45% | 8.17% |
| Global Equity - Factor-Tilted** | 25% | 7.68% | 75% | 7.76% | 7.74% | 0.34% | 7.38% |

Source: PWL Capital; Data Sources: Bloomberg, DFA Returns Web, Robert Shiller, Elroy Dimson, Paul Marsh and Mike Staunton, Triumph of the Optimists: 101 Years of Global Investment Returns, Princeton University Press, 2002; Elroy Dimson, Paul Marsh and Mike Staunton, Credit Suisse Global Returns Yearbook and Sourcebook, 2018, Zurich: Credit Suisse Research Institute, 2021

# 11. Asset Class Expected Volatility

We use the indices discussed in section 6 to estimate the volatility of asset classes. Standard deviations can be quite different when measured with short-term and long-term data. Since it's hard to know whether the recent or the long-term volatility will prevail in the coming 30 years, we adopt a simple average of the 5-year and 20-year standard deviation to estimate future volatility. Table 32 and Table 33 provides the standard deviation calculations for market-cap weighted and factor-tilted portfolios.

### Table 32 - Estimated Volatility of Market-Cap Weighted Asset Classes

| Asset Class | Five-year Standard Deviation | 20-year Standard Deviation | Estimated Standard Deviation |
|---|---|---|---|
| Fixed Income | 5.58% | 4.34% | 4.96% |
| Canadian Equity | 16.72% | 14.29% | 15.50% |
| US Equity | 16.51% | 13.46% | 14.99% |
| International Equity | 12.70% | 13.53% | 13.12% |

Source: PWL Capital; Data Source: DFA Returns Web

### Table 33 - Estimated Volatility of Factor-Tilted Asset Classes

| Asset Class | Five-year Standard Deviation | 20-year Standard Deviation | Estimated Standard Deviation |
|---|---|---|---|
| Fixed Income - Factor-Tilted | 3.52% | 3.78% | 3.65% |
| Canadian Equity - Factor Tilted | 19.69% | 16.11% | 17.90% |
| US Equity - Factor Tilted | 17.20% | 14.39% | 15.79% |
| International Equity - Factor Tilted | 13.90% | 14.56% | 14.23% |

Source: PWL Capital; Data Source: DFA Returns Web

# 12. Expected Correlations

We use the indices discussed in section 6 to estimate the correlation of asset classes. Like volatility, correlations are estimated from a simple 5- and 20-year average.

### Table 34 - Market-Cap Weighted Asset Class Correlations

| | Fixed Income | Canadian Equity | US Equity | International Equity |
|---|---|---|---|---|
| Fixed Income | 1.00 | 0.27 | 0.35 | 0.34 |
| Canadian Equity | 0.27 | 1.00 | 0.73 | 0.73 |
| US Equity | 0.35 | 0.73 | 1.00 | 0.77 |
| International Equity | 0.34 | 0.73 | 0.77 | 1.00 |

Source: PWL Capital; Data Source: DFA Returns Web

### Table 35 - Factor-Tilted Asset Class Correlations

| | Fixed Income - Factor-Tilted | Canadian Equity - Factor-Tilted | US Equity - Factor-Tilted | International Equity - Factor-Tilted |
|---|---|---|---|---|
| Fixed Income - Factor-Tilted | 1.00 | -0.21 | 0.04 | 0.04 |
| Canadian Equity - Factor-Tilted | -0.21 | 1.00 | 0.75 | 0.77 |
| US Equity - Factor-Tilted | 0.04 | 0.75 | 1.00 | 0.80 |
| International Equity - Factor-Tilted | 0.04 | 0.77 | 0.80 | 1.00 |

Source: PWL Capital; Data Source: DFA Returns Web

# 13. Composition of Asset Class Returns

The composition of returns is essential for financial planning. The tax liability in taxable and non-taxable accounts (due to foreign withholding tax) will hinge on the portion of returns assumed to come from interest, Canadian and foreign dividends, and realized and unrealized capital gains.

To estimate the composition of asset class returns, we proceed as follows:

- Establish one or more mutual funds or ETFs representing the passive benchmark for each asset class. These funds are discussed in section 8.
- For fixed income, the average distribution yield is assumed to be the lowest of the underlying fund's current yield and the asset class expected return. Distributions are assumed to be 100% interest income.
- For Canadian equity, distributions are assumed to be 100% Canadian dividends.
- For US and international equity, distributions are assumed to be 100% foreign dividends.
- The balance of expected returns (net of distribution yields) is treated as capital gains.
- We assume a 50%/50% split between unrealized and realized capital gains.

Table 36 and Table 37 illustrate the Composition of asset class expected returns for market-cap weighted and factor-tilted asset classes.

### Table 36 - Composition of Market-Cap Weighted Asset Class Returns

| Asset Class | Expected Return | Current Yield | Interest & Foreign Dividends | Canadian Dividends | Realized Capital Gains | Unrealized Capital Gains |
|---|---|---|---|---|---|---|
| Fixed Income | 4.15% | 2.87% | 2.87% | 0.00% | 0.13% | 1.16% |
| Canadian Equity | 6.95% | 3.10% | 0.00% | 3.10% | 0.38% | 3.46% |
| US Equity | 6.56% | 1.21% | 1.21% | 0.00% | 0.53% | 4.81% |
| International Equity DV + EM | 7.41% | 2.94% | 2.94% | 0.00% | 0.45% | 4.03% |

Source: PWL Capital; Data Sources: Bloomberg, DFA Returns Web, Robert Shiller, Elroy Dimson, Paul Marsh and Mike Staunton, Triumph of the Optimists: 101 Years of Global Investment Returns, Princeton University Press, 2002; Elroy Dimson, Paul Marsh and Mike Staunton, Credit Suisse Global Returns Yearbook and Sourcebook, 2018, Zurich: Credit Suisse Research Institute, 2021

### Table 37 - Composition of Factor-Tilted Asset Class Returns

| Asset Class | Expected Return | Current Yield | Interest & Foreign Dividends | Canadian Dividends | Realized Capital Gains | Unrealized Capital Gains |
|---|---|---|---|---|---|---|
| Fixed Income | 4.09% | 2.04% | 2.04% | 0.00% | 0.21% | 1.85% |
| Canadian Equity | 7.16% | 3.34% | 0.00% | 3.34% | 0.38% | 3.44% |
| US Equity | 7.10% | 1.47% | 1.47% | 0.00% | 0.56% | 5.06% |
| International equity DV + EM | 8.17% | 3.20% | 3.20% | 0.00% | 0.50% | 4.48% |

Source: PWL Capital; Data Sources: Bloomberg, DFA Returns Web, Robert Shiller, Elroy Dimson, Paul Marsh and Mike Staunton, Triumph of the Optimists: 101 Years of Global Investment Returns, Princeton University Press, 2002; Elroy Dimson, Paul Marsh and Mike Staunton, Credit Suisse Global Returns Yearbook and Sourcebook, 2018, Zurich: Credit Suisse Research Institute, 2021

# 14. Portfolio Expected Returns, Standard Deviations and Return Composition

To estimate the expected return, standard deviation and return composition for a variety of stock/bond mixes, we follow a four-step process:

1. We build a fixed-income and a global equity portfolio. The construction of the fixed-income portfolio is discussed in section7. The global equity portfolio uses the regional weights of the DFA Global Equity fund. That fund has a fixed weight of roughly 33% in Canadian stocks, with the balance being allocated to US and international stocks on a market-cap-weighted basis.
2. We estimate, for the fixed income and the global equity portfolios, the expected return, standard deviation, and the portion of returns coming from interest & foreign dividends, Canadian dividends, and realized and unrealized capital gains.
3. We build a series of asset allocation portfolios, starting with 0% equity / 100% fixed income all the way to 100% equity / 0% fixed income.
4. We then estimate the expected return, standard deviation, interest & foreign dividends, Canadian dividends, realized and unrealized capital gains for each asset mix.

## Table 38 - Portfolio Expected Returns, Standard Deviations, and Return Composition - Market Cap Weighted Portfolios

| | | | ESTIMATED RETURN COMPOSITION | | | |
|---|---|---|---|---|---|---|
| **Asset Mix (Equity/Bond)** | **Expected Return** | **Expected Standard Deviation** | **Interest & Foreign Dividends** | **Canadian Dividends** | **Realized Capital Gains** | **Unrealized Capital Gains** |
| 0/100 | 4.15% | 4.96% | 2.87% | 0.00% | 0.13% | 1.16% |
| 5/95 | 4.26% | 5.00% | 2.79% | 0.05% | 0.14% | 1.28% |
| 10/90 | 4.41% | 5.08% | 2.71% | 0.10% | 0.16% | 1.44% |
| 15/85 | 4.58% | 5.30% | 2.63% | 0.15% | 0.18% | 1.62% |
| 20/80 | 4.72% | 5.56% | 2.55% | 0.20% | 0.20% | 1.78% |
| 25/75 | 4.84% | 5.82% | 2.47% | 0.25% | 0.21% | 1.91% |
| 30/70 | 4.97% | 6.16% | 2.39% | 0.30% | 0.23% | 2.06% |
| 35/65 | 5.12% | 6.59% | 2.31% | 0.34% | 0.25% | 2.22% |
| 40/60 | 5.26% | 7.01% | 2.23% | 0.39% | 0.26% | 2.37% |
| 45/55 | 5.39% | 7.44% | 2.15% | 0.44% | 0.28% | 2.51% |
| 50/50 | 5.54% | 7.96% | 2.07% | 0.49% | 0.30% | 2.67% |
| 55/45 | 5.68% | 8.47% | 1.99% | 0.54% | 0.31% | 2.83% |
| 60/40 | 5.81% | 8.99% | 1.91% | 0.59% | 0.33% | 2.98% |
| 65/35 | 5.95% | 9.50% | 1.84% | 0.64% | 0.35% | 3.13% |
| 70/30 | 6.08% | 10.01% | 1.76% | 0.69% | 0.36% | 3.27% |
| 75/25 | 6.23% | 10.61% | 1.68% | 0.74% | 0.38% | 3.43% |
| 80/20 | 6.36% | 11.13% | 1.60% | 0.79% | 0.40% | 3.57% |
| 85/15 | 6.50% | 11.73% | 1.52% | 0.84% | 0.41% | 3.73% |
| 90/10 | 6.62% | 12.24% | 1.44% | 0.89% | 0.43% | 3.87% |
| 95/5 | 6.77% | 12.84% | 1.36% | 0.94% | 0.45% | 4.03% |
| 100/0 | 6.91% | 13.44% | 1.28% | 0.98% | 0.46% | 4.18% |

Source: PWL Capital; Data Sources: Bloomberg, DFA Returns Web, Robert Shiller, Elroy Dimson, Paul Marsh and Mike Staunton, Triumph of the Optimists: 101 Years of Global Investment Returns, Princeton University Press, 2002; Elroy Dimson, Paul Marsh and Mike Staunton, Credit Suisse Global Returns Yearbook and Sourcebook, 2018, Zurich: Credit Suisse Research Institute, 2021

## Table 39 - Portfolio Expected Returns, Standard Deviations, and Return Composition - Factor-Tilted Portfolios

| | | | ESTIMATED RETURN COMPOSITION | | | |
|---|---|---|---|---|---|---|
| **Asset Mix (Equity/Bond)** | **Expected Return** | **Expected Standard Deviation** | **Interest & Foreign Dividends** | **Canadian Dividends** | **Realized Capital Gains** | **Unrealized Capital Gains** |
| 0/100 | 4.09% | 3.65% | 2.04% | 0.00% | 0.21% | 1.85% |
| 5/95 | 4.25% | 3.53% | 2.01% | 0.05% | 0.22% | 1.97% |
| 10/90 | 4.42% | 3.58% | 1.98% | 0.11% | 0.23% | 2.10% |
| 15/85 | 4.59% | 3.72% | 1.95% | 0.16% | 0.25% | 2.23% |
| 20/80 | 4.76% | 4.06% | 1.92% | 0.21% | 0.26% | 2.36% |
| 25/75 | 4.92% | 4.52% | 1.89% | 0.26% | 0.28% | 2.49% |
| 30/70 | 5.07% | 4.98% | 1.87% | 0.32% | 0.29% | 2.60% |
| 35/65 | 5.23% | 5.56% | 1.84% | 0.37% | 0.30% | 2.72% |
| 40/60 | 5.41% | 6.25% | 1.81% | 0.42% | 0.32% | 2.86% |
| 45/55 | 5.55% | 6.82% | 1.78% | 0.48% | 0.33% | 2.97% |
| 50/50 | 5.72% | 7.51% | 1.75% | 0.53% | 0.34% | 3.10% |
| 55/45 | 5.88% | 8.20% | 1.72% | 0.58% | 0.36% | 3.22% |
| 60/40 | 6.04% | 8.89% | 1.69% | 0.64% | 0.37% | 3.35% |
| 65/35 | 6.23% | 9.69% | 1.66% | 0.69% | 0.39% | 3.49% |
| 70/30 | 6.38% | 10.38% | 1.63% | 0.74% | 0.40% | 3.61% |
| 75/25 | 6.54% | 11.07% | 1.60% | 0.79% | 0.41% | 3.73% |
| 80/20 | 6.72% | 11.87% | 1.57% | 0.85% | 0.43% | 3.86% |
| 85/15 | 6.87% | 12.56% | 1.55% | 0.90% | 0.44% | 3.98% |
| 90/10 | 7.04% | 13.37% | 1.52% | 0.95% | 0.46% | 4.12% |
| 95/5 | 7.19% | 14.06% | 1.49% | 1.01% | 0.47% | 4.23% |
| 100/0 | 7.38% | 14.86% | 1.46% | 1.06% | 0.49% | 4.37% |

Source: PWL Capital; Data Sources: Bloomberg, DFA Returns Web, Robert Shiller, Elroy Dimson, Paul Marsh and Mike Staunton, Triumph of the Optimists: 101 Years of Global Investment Returns, Princeton University Press, 2002; Elroy Dimson, Paul Marsh and Mike Staunton, Credit Suisse Global Returns Yearbook and Sourcebook, 2018, Zurich: Credit Suisse Research Institute, 2021

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