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Should my portfolio be on a glide path?

Graham Westmacott CFA
Portfolio Manager
PWL CAPITAL INC.

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For more information about this or other publications from PWL Capital, contact:

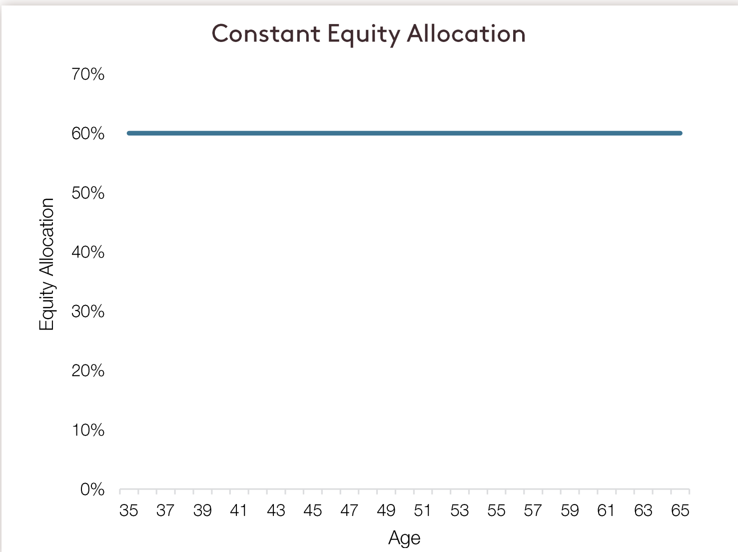
20 Erb Street W., Suite 506, Waterloo, Ontario N2L1T2
Tel 519 880-0888 • 1-877 517-0888 **Fax** 519 880-9997
waterloo@pwlcapital.com • www.pwlcapital.com/white-papers

Introduction

A key decision when designing an investment portfolio is the allocation between equities and bonds. Equities are more risky than bonds so the allocation to equities is the primary contributor to the risk of the total portfolio. An intuitive view is that younger investors can afford to take more risk than older investors so the allocation to equities should decline with time. We consider whether this intuition is supported by the evidence.

Constant Equity Allocation

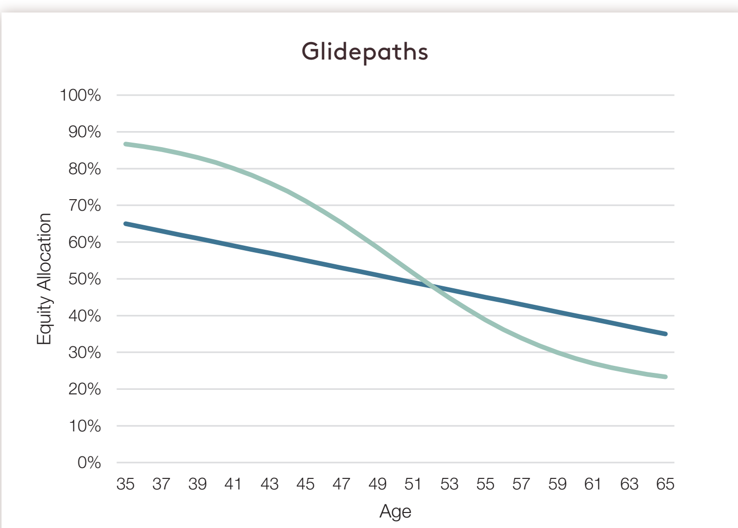
We can choose a constant value for the equity allocation (e.g. 60%). Even though the portfolio grows, continuous rebalancing keeps the proportion allocated to equities constant as illustrated below.



Source: PWL Capital

Time Varying Equity Allocation

An alternative to a constant equity allocation is to allow the proportion of equities to change with the investor's age. This gives rise to what is commonly referred to as a glide path, where the decline may be linear or curved as illustrated below.



Source: PWL Capital

Is a glide path better than a constant equity allocation?

To answer this question we have to be precise about what we mean by “better”. We assume the investor’s objective is to reach a cumulative wealth target with minimum risk, where risk is the spread of possible outcomes as measured by the standard deviation. Consider two investor portfolios, A and B, as illustrated below:

Figure 1 (a,b)

Source: PWL Capital

In Fig 1(a), portfolio A has, on average, accumulated wealth of $200,000, with a standard deviation of $40,000. Over the same investment period, Portfolio B has the same accumulated wealth but a lower standard deviation of $30,000. Portfolio B is preferred over Portfolio A because it achieves the same wealth with greater certainty.

In Fig 1(b) the portfolios both have the same risk, but portfolio A has, on average, a lower accumulated wealth than portfolio B. Portfolio B is preferred because it achieves a higher accumulated wealth with no greater risk.

We define “better” as follows:

A glide path portfolio is better than a constant equity allocation portfolio if they both have the ***same*** cumulative wealth but the glide path portfolio has a ***lower*** risk.

Given that risk increases when equity allocation increases an equivalent statement is:

A glide path portfolio is better than a constant equity allocation portfolio if they both have the ***same*** risk but the glide path portfolio has a ***higher*** cumulative wealth.

The Linear Glide Path with a Lump Sum Investment

Suppose the equity allocation is given by $p(t)$ such that:

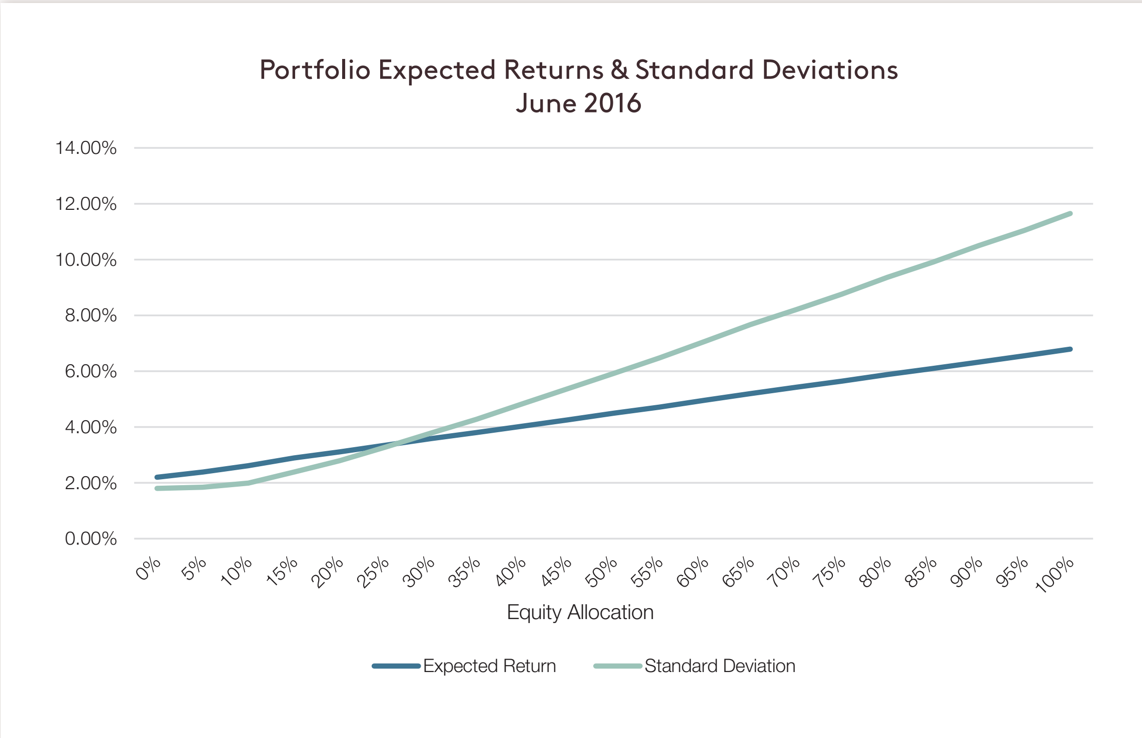
$$p(t) = \frac{(100 - t)}{100} \qquad (1)$$

where t is the age of the investor in years.

For example, a 35 year old would have an asset allocation of 65% equity and an 85 year old would have an asset allocation of 15% equity: the equity allocation decreases with time.

Consider the example of a 35 year old investor who has a single lump sum of $100,000 to invest. We could imagine the investor comes into an inheritance that he wants to pass on to his children, for example. According to the above formula, he reduces his equity allocation every year until age 85. We use a statistical simulator to compute the expected wealth and the risk as measured by the standard deviation. In all our calculations, we use the expected returns and standard deviations for each equity allocation from PWL Capital's expected portfolio returns here.

The chart below shows the expected returns and standard deviations as a function of the portfolio equity allocation.



Source: PWL Capital

In Appendix A, we calculate the constant equity allocation that yields the same cumulative wealth as the glide path. For the situation of a single lump sum investment, this is the average equity allocation over the period which in our case is simply (65%+15%)/2 = 40%.

We compare the decreasing equity glide path and the constant equity allocation in the table below.

	CUMULATIVE AVERAGE WEALTH	STANDARD DEVIATION
Decreasing equity (A)	$723,386	$259,414
Constant 40% equity (B)	$722,303	$244,293
Difference, (B-A)/A	-0.15%	-5.82%

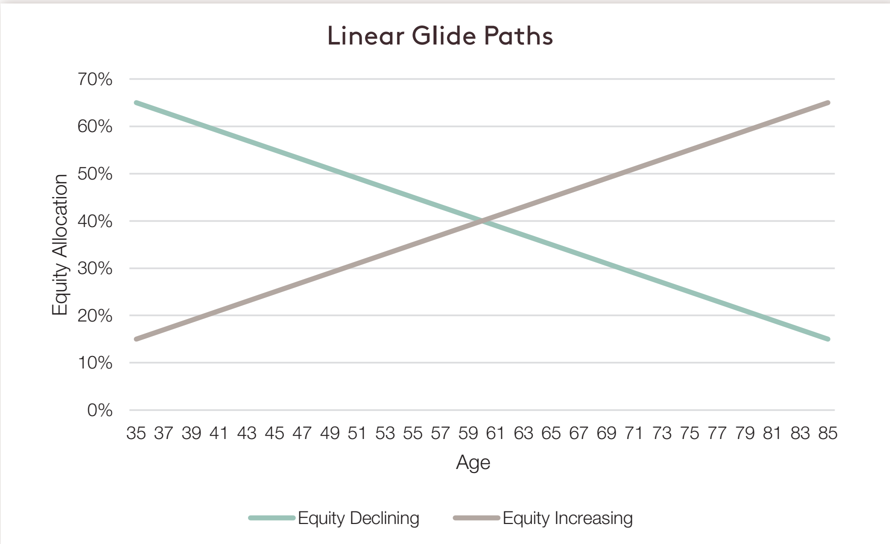
Source: PWL Capital using Returns2 from Dimensional Fund Advisors

As expected, the terminal wealth is very similar, within modelling error[1], but the range of distribution of that wealth, as represented by the standard deviation, is less in the constant equity allocation case. An investor concerned about reaching a terminal wealth would likely prefer the constant equity asset allocation, where the likelihood of achieving their goal is greater.

Suppose we reversed the direction of the glide path so that the investor started with a low equity allocation that increased with age:

$$p(t) = \frac{t}{100} - 0.2 \qquad (2)$$

In this case, the equity allocation at age 35 is 15% and the equity allocation at age 85 is 65%, as illustrated below. Our intuition from the opening paragraph is that equities should decline with age, not increase, so we might anticipate this to be a poor strategy.



Source: PWL Capital

The results are shown below:

	CUMULATIVE AVERAGE WEALTH	STANDARD DEVIATION
Decreasing Equity (A)	$723,386	$259,414
Increasing Equity (B)	$722,599	$257,245
Difference (B-A)/A	-0.04%	-0.08%

Source: PWL Capital using Returns2 from Dimensional Fund Advisors

We observe that increasing equity allocation with time is, within modelling error, no different from decreasing equity allocation with time and that both are worse than a constant 40% equity allocation. These results are a reminder that compounded wealth is a series of annual returns and the sequence of those returns has no impact on the final result. As we will see below, this situation changes when there are contributions during the period.

A more general mathematical analysis[2] shows that we can *always* choose a constant equity allocation, equal to the average equity allocation, which has the same expected wealth but has lower risk than a glide path strategy, irrespective of the shape of the glide path curve.

[1] In the numerical experiments we considered differences of less than +/-1.0% to be insignificant.
[2] See, for example "An Introduction to Computational Finance Without Agonizing Pain" P.A. Forsyth. https://cs.uwaterloo.ca/~paforsyt/agon.pdf

The Linear Glide Path with Regular Savings

Thus far we have considered the impact of asset allocation only on lump sum investing. Next we consider the more realistic scenario where investors are saving regularly.

Consider the example of an investor who starts with nothing but saves $10,000 annually from age 35 to age 65, a savings period of 30 years. The equity allocation declines with age as described by equation (1). The average equity allocation over the savings period is now 50%.

	CUMULATIVE AVERAGE WEALTH	STANDARD DEVIATION
Decreasing Equity (A)	$619,258	$120,944
Constant 50% Equity (B)	$635,996	$137,228
Difference (B-A)/A	2.70%	13.42%

Source: PWL Capital using Returns2 from Dimensional Fund Advisors

A constant equity allocation that is the average equity allocation is no longer a good substitute for the glide path portfolio. The cumulative wealth is now dependent on the timing of the savings.

In Appendix A, we show how to estimate a constant equity allocation that yields the same cumulative wealth as a glide path in the presence of regular savings. The appropriate constant equity allocation is now a weighted average of the equity allocation over the period, using the time invested as the weighting factor. For the example above, we estimate the appropriate constant equity that yields the same cumulative wealth as the glide path is 46%.

In the table below, we compare the results and note that there is no significant difference between the standard deviation of constant equity allocation of 46% and the glide path portfolio.

	CUMULATIVE AVERAGE WEALTH	STANDARD DEVIATION
Decreasing Equity (A)	$619,258	$120,944
Constant 46% Equity (B)	$615,591	$121,207
Difference (B-A)/A	-0.59%	-0.22%

Source: PWL Capital using Returns2 from Dimensional Fund Advisors

For completeness, we repeat the analysis with regular savings and a glide path that increases with time. The equity allocation is 35% at age 35, increasing to 65% at age 65. The constant equity allocation that generates the same cumulative wealth is estimated to be 55%. As expected, the risk is also the same, within modelling error.

	CUMULATIVE AVERAGE WEALTH	STANDARD DEVIATION
Increasing Equity (A)	$659,117	$160,818
Constant 55% Equity (B)	$663,794	$159,302
Difference (B-A)/A	0.71%	-0.94%

Source: PWL Capital using Returns2 from Dimensional Fund Advisors

In conclusion, while glide paths may appear to offer a better approach to accumulating wealth, they can be replaced with a constant equity allocation with very similar outcomes as measured by cumulative wealth and the standard deviation of the cumulative wealth. For lump sum investing, the best constant equity allocation is the simple mean and the risk is lower than using a glide path. For regular savings with a declining equity allocation with age, the best constant equity allocation is slightly less than the simple mean. If the slope of the glide path is reversed, so the equity allocation increases with age, then the best constant equity allocation is slightly higher than the simple mean. Changing the slope of the glide path has only a modest impact on the choice of constant equity allocation.

Implications for Target Date Funds

Target date funds (or lifecycle funds) use the idea of a glide path, so that the equity allocation decreases with age, and are popular in the US (with $763 billion in assets), mainly within defined contribution savings plans. Their presence is growing in Canada with offerings from Fidelity (Clearpath), Blackrock (LifePath) and Vanguard (Target Retirement). They offer the attraction of simplicity: the investor only has to decide their retirement date, put all their savings in the fund with the same target date and the fund manager takes care of the rest. This simplicity can be a real advantage if the alternative is that the investor makes a poor choice of individual funds, or worse, no choice at all. Despite the apparent sophistication of target date funds, a simple balanced fund with a constant equity allocation can perform as well before fees.

Target Date Funds are typically a fund of funds with higher fees. For example the Fidelity target date funds typically have management fees (excluding advisor fees) of 1-1.15%. By comparison, the DFA Global 60% Equity, 40% Fixed Income fund (excluding advisor fees) has a cost of 0.42%. We estimated the impact of a 0.5% additional fee cost on our investor saving $10,000 annually for 30 years to be $53,510 resulting in 8.65% less cumulative wealth.

Our conclusion is that target date funds offer little or no advantage over adhering to a fixed asset allocation if the goal is building maximum wealth with minimum risk before fees. After fees, they are poor value. In the US, target date funds are Qualified Default Investment Alternatives (QDIAs) which means that DC plan contributors who do not specify an investment vehicle for their (and possibly their employer's) contribution can find themselves defaulted into target date funds. The concern is such a default allows the fund providers to take more money in fees without offering any value beyond what could be achieved with a lower cost balanced fund with a constant equity allocation.

A Victory for Constant Asset Allocation?

If an investor is focused solely on achieving a target wealth while minimizing the risk of missing that target, then we conclude that a constant equity asset allocation is likely to be preferable to a glide path strategy, after fees.

As noted above, a lump sum investor could achieve the same outcome whether the equity allocation rises or falls with age. Yet, suggesting increasing asset allocation with age feels like poor advice, which suggests that there are important considerations missing from our model. Two suggestions for missing ingredients are:

1. An investor's wealth at any time influences their subsequent choice of asset allocation. If, for example, the market is so favourable that the wealth target is achieved early, why would the investor continue to risk equity exposure? This suggests that equity allocation should be a function of current wealth, which is dependent on the investor's experience in the market.

2. The cost of meeting a future obligations depends on interest rates. Falling rates means the cost of meeting a future liability rises. If the investor is saving to match a future liability (e.g. a pension requirement) then the end wealth also needs to change to match the changing liability as interest rates change.

Both these observations suggest that asset allocation should not be pre-determined but allowed to adapt to better meet investor's objectives. In a paper on retirement portfolio design, we allow the overall asset allocation to emerge from a continuously updated assessment of the future income liability and the current market portfolio.

More generally, recent studies[3] suggest that adaptive schemes that use both the time to the target wealth and the current portfolio value can achieve the same cumulative wealth as a constant equity allocation portfolio with significantly lower cumulative and downside risk. Glide paths are here to stay, but as an output rather than an input to the portfolio design.

[3] https://cs.uwaterloo.ca/~paforsyt/long_term.pdf (Working Paper)

Appendix A

Constant Savings on a Glidepath

Consider a series of regular savings over a period of T years,

$$I = \{I_1, I_2, I_3, \ldots, I_T\}$$

Each savings grows according to the return in the year which is a function of the equity allocation in that year. In year t:

$$r_t = r(p_t)$$

where r is the return in period t and p_t is the equity allocation in period t.

The wealth at the end of period 1 is

$$W_{1,1} = I_1 \{1 + r(p_1)\}$$

Where the savings I_1 is assumed to be at the start of the period.

At period t

$$W_{1,t} = I_1 \{(1 + r(p_1))\ (1 + r(p_2))(1 + r(p_3))\ \ldots\ (1 + r(p_t))\}$$

where $W_{1,t}$ is the wealth accumulated from savings I at the end of period t.

The total wealth from savings I_i is:

$$W_{i,T} = I_i \prod_{j=1}^{T} (1 + r(p_j))^j$$

Thus the total wealth from all savings,

$$W_T = \sum_{i=1}^{T} W_{i,T} \qquad (3)$$

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Single Lump Sum Investment

We consider the cumulative wealth from a single lump sum, I invested to the end of period T.

$$W_T = I \prod_{j=1}^{T} \left(1 + r(p_j)\right)^j$$

We wish to determine a constant equity asset allocation, p^* that yields the same cumulative wealth.

Thus

$$W_T = I \left(1 + r(p^*)\right)^T$$

where I is the initial lump sum invested.

This requires

$$\left(1 + r(p^*)\right)^T = (1 + r_1)\,(1 + r_2)\,\ldots\,(1 + r_T)$$

Where for brevity

$$r_t = r(p_t)$$

So

$$Tln\,\left(1 + r(p^*)\right) = \ln(1 + r_1) + \ln(1 + r_2) \ldots \ln(1 + r_T)$$

A Taylor expansion around $r_i = 0$ yields the approximation that $r(p^*)$ is the mean

$$r(p^*) = \frac{1}{T} \sum_{i=1}^{T} r_i$$

Constant Savings

Assume a constant savings rate, I. We seek a constant equity allocation, p', such that

$$W_T = I\left\{(1 + r(p'))^T + (1 + r(p'))^{T-1} \ldots (1 + r(p'))\right\}$$

For $r(p') << 1$

$$W_T = I\left\{T + r(p')\sum_{i=1}^{T} i\right\} + 0(r^2) \qquad (4)$$

From (1) above:

$$\begin{aligned} W_T = I\{&(1 + r_1)(1 + r_2)(1 + r_3) \ldots (1 + r_T) \\ &+ (1 + r_2)(1 + r_3) \ldots (1 + r_T) \\ &+ (1 + r_3) \ldots (1 + r_T) \\ &\ldots \\ &+ (1 + r_T)\} \end{aligned}$$

For $r_i << 1,$

$$W_T = I(\mathrm{T} + r_1 + 2r_2 + 3r_3 + \cdots + \mathrm{T}r_T) + 0(r^2)$$

$$W_T = I\left\{T + \sum_{i=1}^{T} i r_i\right\} \qquad (5)$$

From (4) and (5)

$$r(p')\sum_{i=1}^{T} i = \sum_{i=1}^{T} i r_i$$

So we get the weighted mean for $r(p')$:

$$r(p') = \sum_{i=1}^{T} i r_i \Big/ \sum_{i=1}^{T} i$$

Graham Westmacott CFA
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PWL CAPITAL INC.

gwestmacott@pwlcapital.com
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PWL Waterloo

20 Erb St. W, Suite 506
Waterloo, Ontario N2L 1T2

T 519.880.0888
1.877.517.0888

F 519.880.9997
waterloo@pwlcapital.com

www.pwlcapital.com/Waterloo

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