How to Calculate your Portfolio's Rate of Return

A guide to better understanding and calculating various portfolio returns

Justin Bender, CFP
Investment Advisor
PWL CAPITAL
Toronto, Ontario

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This report was written by Justin Bender, PWL Capital Inc. The ideas, opinions, and recommendations contained in this document are those of the author and do not necessarily represent the views of PWL Capital Inc.

For more information about this or other publications from PWL Capital, contact:
3400 de Maisonneuve Ouest, Bureau 1501, Montréal, Québec H3Z 3B8
Tel 514-875-7566 • 1-800-343-7566 x264 **Fax** 514-875-9611
capital@pwlcapital.com

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Introduction

In an ideal world, clients would be updated annually with their portfolio's rate of return, enabling them to compare their performance to the appropriate benchmarks. In reality, some advisors may not be providing these results to their clients, so how should investors proceed with this daunting task? Which return measurement should be used, and what are the differences between them? How should they account for contributions to and withdrawals from their portfolio? Listed below are four of the most commonly calculated returns, which will be discussed in detail in the examples that follow.

- Money-Weighted Rate of Return (MWR or IRR)
- Linked Internal Rate of Return (LIRR)
- Modified Dietz Rate of Return (MDR)
- True Time-Weighted Rate of Return (TWR)

The money-weighted rate of return, linked internal rate of return, and the Modified Dietz rate of return can all differ substantially from the true time-weighted return depending largely on a combination of the following three variables:

1. The timing of the cash flows
2. The size of the cash flows relative to the size of the portfolio
3. The volatility of the portfolio's market value

Suppose an investor has a portfolio with a total market value on December 31, 2010 (including accrued income) of $250,000. In Chart 1.1, the month-end market values for the first quarter of 2011 are included, as well as the dates and amounts of any contributions (positive external cash flows) and withdrawals (negative external cash flows), before and after each cash flow occurred.

Chart 1.1:

Date	Market Value (MV)	Cash Flow (CF)	MV After Cash Flow
December 31, 2010	$250,000		
January 31, 2011	$255,000		
February 14, 2011	$257,000	+ $25,000	$282,000
February 28, 2011	$288,000		
March 10, 2011	$293,000	- $10,000	$283,000
March 31, 2011	$285,000		

Throughout these exercises, you will need to continually refer back to Chart 1.1, so keep it handy as you work through each calculation. As a side note, *accrued income (i.e. accrued interest and declared dividends) will need to be included in your market value inputs (i.e. MV_0, MV_1) if your portfolio contains individual bonds, GICs, or stocks.* While this is not an impossible task, it is easier said than done. I would consider finding an advisor who will calculate this for you accurately and efficiently.

How to Calculate your Money-Weighted Rate of Return

(Internal Rate of Return, or IRR)

The money-weighted rate of return (MWR), more commonly referred to as the internal rate of return (IRR), is the average return that makes the net present value of the sum of all cash flows during the measurement period equal to 0. For this example, we will assume all contributions to the portfolio (including the initial market value of the portfolio, MV0) to be positive cash flows, and all withdrawals from the portfolio (including the ending market value of the portfolio, MV1) to be negative cash flows.

This method can be useful for calculating the rate of return when there have been only small external cash flows during the measurement period, relative to the size of the portfolio. As it assumes all cash flows receive the same rate of return while invested, its return can differ substantially from the true time-weighted rate of return when large cash flows occur during periods of significantly fluctuating portfolio values.

Equation 1:

$$0 = \frac{MV_0}{(1 + IRR)^{w(0)}} + \sum\left[\frac{CF_i}{(1 + IRR)^{w(i)}}\right] + \frac{MV_1}{(1 + IRR)^{w(1)}}$$

where:
MV_0 = full market value of the portfolio, including accrued income, at the beginning of the period
MV_1 = full market value of the portfolio, including accrued income, at the end of the period
IRR = internal rate of return
$w(i) = D_i/CD$
D_i = the number of calendar days from the beginning of the period that cash flow CFi occurs
CD = the total number of calendar days in the measurement period
CF_i = cash flow i

Source: CFA Institute adapted by PWL Capital Inc.

Using the above equation and the values from Chart 1.1 (and through trial-and-error), investors could begin by plugging in values for the IRR until the right-hand side of the equation equals approximately 0.

$$0 \approx \frac{\$250{,}000}{(1 + IRR)^{(0/90)}} + \frac{\$25{,}000}{(1 + IRR)^{(45/90)}} + \frac{(-\$10{,}000)}{(1 + IRR)^{(69/90)}} + \frac{(-\$285{,}000)}{(1 + IRR)^{(90/90)}}$$

where: CD = 90 = the total number of calendar days during the first quarter of 2011

Example 1.1:

IRR Guess1 = 10%; → Equation = $5,450 ✗		IRR Guess6 = 7.5%; → Equation = -$465 ✗	
IRR Guess2 = 5%; → Equation = -$6,664 ✗		IRR Guess7 = 7.6%; → Equation = -$223 ✗	
IRR Guess3 = 6%; → Equation = -$4,149 ✗		IRR Guess8 = 7.7%; → Equation = $19 ✗	
IRR Guess4 = 7%; → Equation = -$1,681 ✗		IRR Guess9 = 7.69%; → Equation = -$6 ✓	
IRR Guess5 = 8%; → Equation = $740 ✗		IRR Guess10 = 7.68%; → Equation = -$30 ✗	

Through trial-and-error, we find that **7.69%** is the closest internal rate of return (within 2 decimal places) that makes the net present value of the sum of all cash flows during the measurement period approximately equal to 0.

As this method is tiresome and no longer necessary with the advent of computers, investors interested in calculating their portfolio's money-weighted rate of return can save themselves a lot of time and effort by visiting Weigh House Investor Services' online calculator and entering their beginning and ending portfolio values (including accrued income) for the measurement period, as well as contribution and withdrawal amounts and the date that each cash flow occurred. This method requires the least amount of portfolio information relative to the other calculation methods. However, it may result in the least accurate return, especially over longer measurement periods. The online calculator only allows the investor to input 12 cash flows per measurement period. For investors with more than 12 cash flows in their measurement period, the linked internal rate of return example will provide a solution for this common issue.

Example 1.2:

Start Date (mm/dd/yyyy)	12/31/2010
Value of Portfolio on Start Date	250000
End Date (mm/dd/yyyy)	03/31/2011
Value of Portfolio on End Date	285000

Contributions and Withdrawals between the Start Date and the End Date

Date (mm/dd/yyyy)	Funds Added to Portfolio	Funds Removed from Portfolio
02/14/2011	25000	
03/10/2011		10000

Average rate of return 7.69%

Source: Weigh House Investor Services

How to Calculate your Linked Internal Rate of Return

(Approximate Time-Weighted Rate of Return)

By geometrically linking monthly money-weighted rates of return, an approximate time-weighted rate of return, appropriately called the linked internal rate of return (LIRR), can be calculated. For measurement periods of one year or less, geometric linking involves converting the monthly money-weighted returns to relative form (1 + MWRi), multiplying them together, and subtracting 1 from the result.

Equation 2:

$$LIRR = \left[(1 + MWR_1) \times (1 + MWR_2) \times \ldots \times (1 + MWR_i)\right] - 1$$

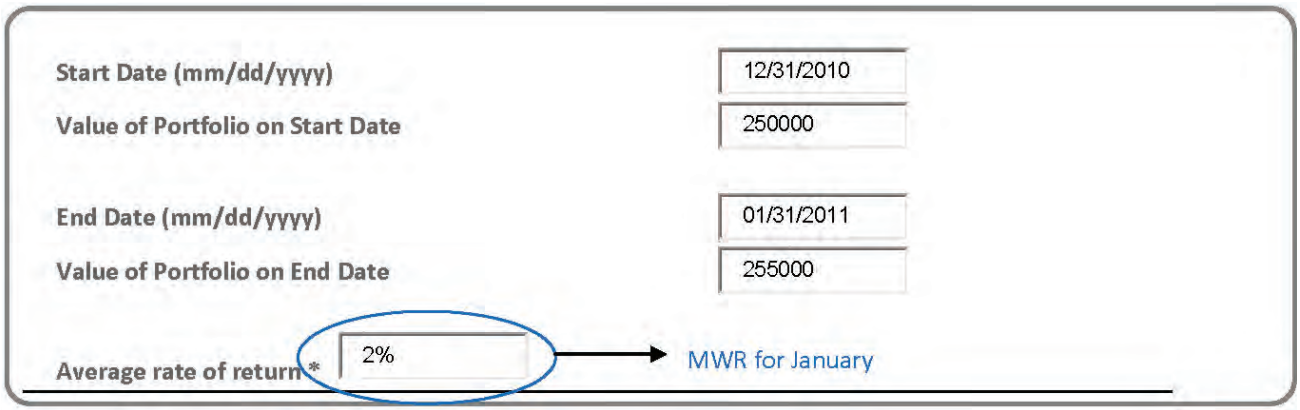
where:
LIRR = linked internal rate of return
MWR_i = series of monthly money-weighted rates of return
$(1 + MWR_i)$ = monthly money-weighted return converted to relative form

Source: CFA Institute adapted by PWL Capital Inc.

As long as the portfolio has not experienced large external cash flows in combination with volatile portfolio fluctuations during the monthly measurement period, the linked internal rate of return should fall fairly close to the true time-weighted rate of return.

In our example, we can once again use the Weigh House Investor Services' online calculator, this time individually calculating the monthly money-weighted returns and geometrically linking them together afterwards to create an approximate time-weighted rate of return (this method may also be useful for investors with more than 12 cash flows in a given measurement period – by calculating monthly returns and geometrically linking the results, the online calculator now allows for 12 cash flows per month). The downside of using this approach is that month-end portfolio values are now required throughout the entire measurement period.

Example 2:



Source: Weigh House Investor Services

Start Date (mm/dd/yyyy)	01/31/2011
Value of Portfolio on Start Date	255000
End Date (mm/dd/yyyy)	02/28/2011
Value of Portfolio on End Date	288000

Contributions and Withdrawals between the Start Date and the End Date

Date (mm/dd/yyyy)	Funds Added to Portfolio	Funds Removed from Portfolio
02/14/2011	25000	

Average rate of return * 2.99% → MWR for February

Source: Weigh House Investor Services

Start Date (mm/dd/yyyy)	02/28/2011
Value of Portfolio on Start Date	288000
End Date (mm/dd/yyyy)	03/31/2011
Value of Portfolio on End Date	285000

Contributions and Withdrawals between the Start Date and the End Date

Date (mm/dd/yyyy)	Funds Added to Portfolio	Funds Removed from Portfolio
03/10/2011		10000

Average rate of return * 2.49% → MWR for March

Source: Weigh House Investor Services

Geometric Linking of Monthly Returns

$$\text{LIRR} = \left[(1 + 0.0200) \times (1 + 0.0299) \times (1 + 0.0249)\right] - 1 = \mathbf{7.67\%}$$

How to Calculate your Modified Dietz Rate of Return

(Approximate Time-Weighted Rate of Return)

The Modified Dietz rate of return (MDR) is currently the method used by PWL Capital to calculate our clients' portfolio returns. It attempts to estimate a true time-weighted rate of return by weighting each cash flow by the proportion of the measurement period it is present or absent from the portfolio. Monthly Modified Dietz rates of return are usually calculated and geometrically linked together, similar to the process we saw above with the Linked Internal Rate of Return. Since the portfolio is valued monthly and not daily, the Modified Dietz rate of return can still differ widely from the true time-weighted rate of return if large external cash flows occur during volatile monthly measurement periods.

Equation 3:

$$ModDietz = \frac{MV_1 - MV_0 - CF}{MV_0 + \sum (CF_i \times W_i)}$$

where:

MV_1 = full market value of the portfolio, including accrued income, at the end of the period

MV_0 = full market value of the portfolio, including accrued income, at the beginning of the period

$\sum (CF_i \times W_i)$ = the sum of each cash flow multiplied by its weight

$CF = \sum CF_i$

$W_i = (CD - D_i)/CD$

CD = the total number of calendar days in the period

D_i = the number of calendar days from the beginning of the period that cash flow CF_i occurs

Note: This formula assumes cash flows occur at the end of the day.

Source: CFA Institute

Example 3:

Monthly period	W_i	*ModDietz*	
January 1, 2011 to January 31, 2011		$= \frac{\$255,000 - \$250,000}{\$250,000}$	= 2.00%
February 1, 2011 to February 28, 2011	$= \frac{28 - 14}{28} = 0.5$	$= \frac{[\$288,000 - \$255,000 - (\$25,000)]}{[\$255,000 + (\$25,000 \times 0.5)]} = \frac{\$8,000}{\$267,500}$	= 2.99%
March 1, 2011 to March 31, 2011	$= \frac{31 - 10}{31} = 0.6774$	$= \frac{[\$285,000 - \$288,000 - (- \$10,000)]}{[\$288,000 + (- \$10,000 \times 0.6774)]} = \frac{\$7,000}{\$281,226}$	= 2.49%
January 1, 2011 to March 31, 2011	*Geometric Linking* →	= [(1.0200) × (1.0299) × (1.0249)] – 1	**= 7.67%**

How to Calculate your True Time-Weighted Rate of Return

(Daily Valuation Method)

Lastly, we arrive at the Holy Grail of portfolio performance measurement; the true time-weighted rate of return (TWR). Although this is arguably the most accurate portfolio return in most situations, it requires daily portfolio valuations whenever an external cash flow (i.e. a contribution or withdrawal) occurs. Months in which external cash flows occur are divided into sub-periods, each with its own total return calculation. These sub-period returns are then geometrically linked together to obtain the true time-weighted rate of return over the measurement period.

Equation 4:

$$TWR = \left[(1 + r_{t,1}) \times (1 + r_{t,2}) \times \ldots \times (1 + r_{t,n})\right] - 1$$

where:

$$r_{t,n} = \frac{MV_1 - MV_0}{MV_0}$$

Source: CFA Institute adapted by PWL Capital Inc.

Example 4:

Sub-period	$r_{t,n}$	*TWR*	
December 31, 2010 to January 31, 2011	$= \frac{\$255{,}000 - \$250{,}000}{\$250{,}000} = 2.00\%$		$= 2.00\%$
January 31, 2011 to February 14, 2011 *(before cash flow)*	$= \frac{\$257{,}000 - \$255{,}000}{\$255{,}000} = 0.78\%$		
February 14, 2011 *(after cash flow)* to February 28, 2011	$= \frac{\$288{,}000 - \$282{,}000}{\$282{,}000} = 2.13\%$		
January 31, 2011 to February 28, 2011	*Geometric Linking*	$= [(1.0078) \times (1.0213)] - 1$	$= 2.93\%$
February 28, 2011 to March 10, 2011 *(before cash flow)*	$= \frac{\$293{,}000 - \$288{,}000}{\$288{,}000} = 1.74\%$		
March 10, 2011 *(after cash flow)* to March 31, 2011	$= \frac{\$285{,}000 - \$283{,}000}{\$283{,}000} = 0.71\%$		
February 28, 2011 to March 31, 2011	Geometric Linking	$= [(1.0174) \times (1.0071)] - 1$	$= 2.46\%$
December 31, 2010 to March 31, 2011	*Geometric Linking*	$= [(1.0200) \times (1.0293) \times (1.0246)] - 1$	**$= 7.57\%$**

Conclusions

When large contributions are made prior to a period of relatively good performance, or large withdrawals are made prior to a period of poor performance, the money-weighted rate of return, linked internal rate of return, and Modified Dietz rate of return will all ***overstate portfolio performance***, relative to the true time-weighted weight of return.

When large contributions are made prior to a period of relatively poor performance, or large withdrawals are made prior to a period of good performance, the money-weighted rate of return, linked internal rate of return, and Modified Dietz rate of return will all ***understate portfolio performance***, relative to the true time-weighted rate of return.

Shortening the measurement periods to at least monthly can help better approximate the true time-weighted rate of return. If you are currently calculating your money-weighted rate of return (MWR) over an annual measurement period, consider calculating the MWR over monthly intervals, and geometrically linking the results.

Weigh House Investor Services' online calculator can be used to calculate monthly money-weighted rates of return (MWR). The investor can then geometrically link the monthly returns together to obtain a more accurate linked internal rate of return (LIRR) for the year. This will also allow the investor to input up to 12 external cash flows per month.

The true time-weighted rate of return is the best method in most situations for gauging manager performance, as portfolio returns are assessed independent of any external cash flows (which are usually initiated by the client and are therefore, out of the manager's control).

To ensure accuracy, ***accrued income must be included in all market values*** when calculating your rate of return. If the portfolio values available to you do not include accrued income (i.e. your portfolio consists of more than just mutual funds and exchange traded funds, whose net asset values at month-end should already include accrued interest and declared dividends), professional assistance is recommended.

If interest and dividends are being received in cash instead of remaining in the portfolio, each of these cash flow withdrawals will need to be accounted for in your calculations.

It is the overall portfolio return that is important, not the individual account returns. Many clients get frustrated with the returns of their registered accounts (which normally hold the majority of their less volatile fixed income investments) during an upward trending stock market, and experience the same frustration with their non-registered accounts (which normally hold the majority of their volatile equity investments) during a downward trending stock market.

If you have U.S. dollar accounts, the month-end portfolio values will need to be converted to Canadian dollars. Visit the Bank of Canada's US$/CAN$ closing rate summary to obtain the month-end closing exchange rates for the past 12 months (see chart below). Simply take your month-end portfolio value in U.S. dollars, and divide it by the exchange rate shown for the applicable month. Using the chart below, a $250,000 U.S. dollar portfolio value at the end of December 2010 would be equivalent to a $251,357.33 Canadian dollar portfolio value ($250,000 / 0.9946 = $251,357.33).

US$/CAN$ closing rate summary

Monthly closing, past 12 months	
February 2011	0.9714
January 2011	1.0015
December 2010	0.9946
November 2010	1.0266
October 2010	1.0202
September 2010	1.0290
August 2010	1.0665
July 2010	1.0283
June 2010	1.0646
May 2010	1.0435
April 2010	1.0158
March 2010	1.0158
February 2010	1.0525

Source: Bank of Canada

If you have external cash flows into and out of a U.S. dollar account where the financial institution has not provided the Canadian dollar equivalent, the cash flow amounts will also need to be converted to Canadian dollars as of the date the cash flow occurred. Using the Bank of Canada's 10-year exchange rate look-up, select a single date (March 10, 2011), then a currency [U.S. dollar (close)], and then click "Get Rates". Following the same method as above, a $10,000 withdrawal in U.S. dollars is converted to a $10,250.10 withdrawal in Canadian dollars ($10,000 / 0.9756 = $10,250.10).

BANK OF CANADA
BANQUE DU CANADA

	Currency	ISO 4217
	U.S. dollar (close)	USD
Date	1 USD -> CAD	1 CAD -> USD
10/03/2011	**0.9756**	1.0250

Source: Bank of Canada

With all the complexities involved in calculating a portfolio's rate of return, it is recommended that the average client find an advisor or financial institution that has the ability and resources to calculate this for them.

Appendix

Sub-period	Money-Weighted Rate of Return (MWR or IRR)	Linked Internal Rate of Return (LIRR)	Modified Dietz Rate of Return (MDR)	True Time-Weighted Rate of Return (TWR)
December 31, 2010 to January 31, 2011	n/a	2.00%	2.00%	2.00%
January 31, 2011 to February 28, 2011	n/a	2.99%	2.99%	2.93%
February 28, 2011 to March 31, 2011	n/a	2.49%	2.49%	2.46%
December 31, 2010 to March 31, 2011	**7.69%**	**7.67%**	**7.67%**	**7.57%**

Kathleen Clough, CIM, CFP, R.F.P.
Associate Portfolio Manager
kclough@pwlcapital.com
pwlcapital.com/KathleenClough/Blog

Justin Bender, CFP, B.Comm.
Investment Advisor
jbender@pwlcapital.com
pwlcapital.com/Justin-Bender-blog

PWL Capital Inc.

With offices in Montreal, Ottawa, Rivière-du-Loup, Toronto and Waterloo and a combined staff of more than 40 financial professionals, PWL Capital Inc. offers integrated wealth management solutions for a successful investment experience.

Our PWL Toronto office is headed by **Kathleen Clough**, *Associate Portfolio Manager*, she focuses on developing strong, caring bonds with all her clients. The customized solutions she develops for them draw on the depth of her 25 years' professional experience. Kathleen takes some of the worry off your shoulders, organizes your financial affairs and ensures that smart financial decisions are made for your investment goals.

The PWL Toronto team will work closely with you to build a custom financial strategy to reduce your risk and build value.

For more information, please visit **www.pwlcapital.com/Toronto**
T 416 203.0067
1 866 242.0203

Montreal

3400 de Maisonneuve O.
Suite 1501
Montreal, Quebec
H3Z 3B8

Tel 514-875-7566
1 800-343-7566

Fax 514-875-9611

capital@pwlcapital.com

Ottawa

265 Carling Avenue
Suite 100
Ottawa, Ontario
K1S 2E1

Tel 613-237-5544
1 800-230-5544

Fax 613-237-5949

ottawa@pwlcapital.com

Toronto

3 Church Street
Suite 601
Toronto, Ontario
M5E 1M2

Tel 416-203-0067
1 866-242-0203

Fax 416-203-0544

toronto@pwlcapital.com

Waterloo

20 Erb St. West
Suite 506
Waterloo, Ontario
N2L 1T2

Tel 519-880-0888
1 877-517-0888

Fax 519.880.9997

waterloo@pwlcapital.com

Rivière-du-Loup

494 rue Lafontaine
PO Box 397
Rivière-du-Loup, Quebec
G5R 3Y9

Tel 418-862-5643
1 800-774-7418

Fax 418-862-3585

rdl@pwlcapital.com